

Single-vendor single-buyer with integrated transport-inventory system: Models and heuristics in the case of perishable goods

Simone Zanoni, Lucio Zavanella *

Dipartimento d'Ingegneria Meccanica Industriale, Facoltà di Ingegneria, Università degli Studi di Brescia Via Branze, 38-25123 Brescia, Italy

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Abstract

We consider the problem of shipping a set of products from a single origin (the vendor) to a common destination (the buyer) with the objective of minimizing the sum of the inventory and transportation costs, when a set of shipping frequencies is given and products are assumed to be perishable. We provide a mixed integer linear programming model for the problem and propose the modification of known heuristic algorithms to solve it. Extensive computational results show how some of the modified heuristics are extremely efficient and effective.

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1. Introduction

A two-stage supply chain is analysed, where each stage consists of a single node: the logistic network is characterised by a single origin (the production node), which supplies the final products to a single destination (the distribution node) at a constant rate. The distribution node buys the products to meet an external demand, at the same given rate. The objective is to satisfy the final demand, defining the shipping frequencies for each product type so as to minimise the total sum of transportation and inventory costs. Shipments are carried out on the basis of a given set of frequencies.

The minimisation of the supply-chain inventory and transportation costs has been widely tackled in literature, taking into account the different features of real situations. Two main lines of research can be identified: they differ in the type of model proposed to solve the problem.

In the first category, we can include the models that describe the problem by means of a non-linear objective function, typically for the lot-size optimisation in the production and distribution node. Examples are

* Corresponding author. Tel.: +39 030 3715581; fax: +39 030 3702448.
E-mail address: zavanell@ing.unibs.it (L. Zavanella).

Crowston, Wagner, and Williams (1973), Schwarz and Schrage (1975), the Joint Economic Lot Size model (Goyal, 1977), Schwarz and Graves (1977), Lu (1995), Goyal (1995), Hill (1997, 1999, 2000), Viswanathan (1998), Goyal and Nebebe (2000), Hoque and Goyal (2000). In their contributions, the Authors concentrate on determining the optimal solution in a closed form. Generally, they determine the optimal lot-size for lower manufacturing set-up and holding costs, but they do not take into account important real features such as transportation costs.

In the second category, we can include the studies that adopt a linear formulation of the problem. The main advantage of these models is the LP-solvability, which allows the Authors to consider additional real features such as transportation costs. Examples are Speranza and Ukovich (1994, 1996), Bertazzi and Speranza (1999a, 1999b) and Bertazzi et al. (2000), where exact, approximate or heuristic algorithms are proposed.

In contrast to several models presented in literature, which assume that items can be stored indefinitely to meet future demand, in this paper we consider perishable products. Many items in practice are perishable. Fruit, vegetables, meat, foodstuffs, perfumes, alcohol, gasoline, photographic films, etc., are just some examples where deterioration may typically occur during their normal storage period. If the deterioration rate is significant, its impact on the inventory-transportation system cannot be ignored. Perishability refers to the possible physical deterioration of a product and is different from its obsolescence, which generally refers to cases where the demand for the product becomes negligible after a certain time. Perishable products can be subdivided into two categories, according to the characteristic associated with their life span:

- I. *products with fixed life span*, when products maintain their usefulness for a fixed period of time and then must be deemed useless;
- II. *products with a variable life span*, if their usefulness diminishes over time, according to the product age.

The first work on perishability is attributable to Ghare and Schrader (1963). The authors pointed out that inventory decay could exert a significant impact on the overall inventory costs, while considerable cost savings could be achieved if the inventory analysis took the inventory decay into consideration. The first comprehensive review on perishable products can be found in Nahmias (1982). More recently, Goyal and Giri (2001) propose an excellent review of the classification of perishable products and the policies for managing them.

We will focus on perishable products with a fixed life span. The starting point of our work is the mixed-integer linear programming model with given set of frequencies proposed by Speranza and Ukovich (1994). Our aim is to introduce new features, which have been only partially considered by other authors, such as increasing inventory costs through the different nodes of the supply chain (see for instance Bertazzi & Speranza, 1999a). In this case, variable inventory costs are considered in the different nodes of the logistic chain, but computational results are not provided. Furthermore, the perishability of the products will be considered in order to accurately measure its impact on costs, using extensive computational analysis.

In particular, we consider a single supplier furnishing just one company with a given set of products. The supplier produces a set of products, any of which manufactured by a dedicated production line. Production rates of each product are equal to their final demand rates. In other words, production at one node and demand at the other occurs continuously over time and at a constant rate. Thanks to this assumption, set-up costs may be ignored. The destination node receives the products from the sole supplier and sells them in the final market, without stock-out possibility. We do not consider the cost of order issuing. Both the vendor at the origin and the buyer at the destination bear the holding costs, but at different rates. Typically, the holding cost per unit of product in the first node is lower than that in the second node: every product going from the first node to the second one undergoes a series of activities that increase its value. An unlimited fleet of vehicles with fixed capacity is available for the transportation. As in Bertazzi, Speranza, and Ukovich (2000) the vehicle capacity is expressed as volume, even if it could be defined analogously in terms of weight. Shipments are performed by vehicles according to a given set of frequencies set by the internal organisation or by the external contract-based constraints. The objective is to determine a periodic policy of production, supply and transportation that minimizes the sum of the corresponding total costs on an infinite time horizon, while satisfying final demand.

The problem is NP-hard, given that it is a generalisation of the problem shown to be NP-hard in Speranza and Ukovich (1996).

The paper is organised as follows. Section 2 describes the problem, with its mathematical formulation. Section 3 introduces some simple heuristics, while detailed computational results are shown in Section 4. Finally, the formal proof of all the propositions is reported in Appendix A.

2. Problem formulation

We consider the problem where a set $I = \{1, 2, \dots, |I|\}$ of perishable products have to be shipped from a common origin (vendor node, i.e. node A) to a common destination (buyer node, i.e. node B). Products are made available in A and requested at B at the same constant rate. Shipments are performed on the basis of a given set of frequencies.

The following assumptions are made:

- inventory cost per time unit of each type of product is different at the two nodes A and B ;
- transportation costs are independent from the quantity transported, i.e. we assume a fixed cost for each vehicle and shipment;
- production rate is equal to demand rate;
- neither set-up costs at node A nor order costs at node B are considered since, for each product type, the production rate is equal to the demand rate. The absence of set-up costs can be easily justified;
- inventory stock-out cannot occur over the time horizon;
- an unlimited number of homogeneous vehicles with given capacity is available;
- products are perishable: they have to be produced, transported and sold by an expiry date which is different for each product type. This date may represent either the real expiry date or a deadline, nearer than the expiry date, by which the product type is to be sold in order to guarantee the final customer a product with a sufficient residual life;
- products are assumed to be processed in each node according to a FIFO (First In First Out) rule.

Table 1
Notation

| | |
|---------------------------------|------------------------------------------------------------------------------------------------|
| H | Time horizon |
| T | Set of discrete time instants, $\{0, 1, \dots, H - 1\}$ |
| t | Time instant ($t \in T$) |
| I | Set of product indices $\{1, 2, \dots, I \}$ |
| i | Index of product ($i \in I$) |
| J | Set of frequency at which it is possible to carry out a shipment |
| j | Index of shipment frequency ($j \in J$) |
| f_j | j th shipment frequency ($j \in J$) |
| t_j | Period corresponding to frequency f_j $t_j = 1/f_j$ |
| q_i | Rate at which product i is made available at the origin and demanded at the destination |
| v_i | Volume per unit of product i |
| d_i^A (d_i^B) | Initial inventory of product i at node A (B) |
| s_{it} | Quantity of product i shipped at time t from the origin (A) to the destination (B) |
| y_j | Number of vehicles used at frequency f_j |
| r | Transportation capacity of each vehicle |
| c | Transportation cost per journey performed |
| I_i^A (I_i^B) | Inventory of product i at time t and node A (B) |
| \bar{I}_i^A (\bar{I}_i^B) | Average inventory of product i at node A (B) |
| h_i^A (h_i^B) | Inventory cost of one unit of product i per time unit in node A (B) |
| ts_i | Perishable time of product i |
| J_t | Set of frequency at which it is possible to carry out a shipment at time t |
| K | Set of all shipment time instants |
| $K_t = \{k \in K: k \leq t\}$ | Set of shipping time instants lower than or equal to t |
| x_{ij} | Fraction of product i shipped at frequency f_j |

The following notation will be used to define the problem.

The time horizon is discretized and each event may happen only at definite time instants, i.e. shipments may happen only according to the given set of frequencies. A shipment can be carried out only on the basis of a given frequency f_j , $j \in J$, and each frequency is such that the corresponding period t_j , between two consecutive shipments, is integer. As a consequence, we can assume that all the t_j values are multiples of a minimal time unit between shipments (say 1). Consequently, the time horizon H is equal to the *minimum common multiplier* of all the periods associated with the given frequencies. Moreover, the frequencies are synchronized in 0: for each frequency f_j , a set $S_j \subset T$ has been defined, which is the set of time instants at which a shipment can be carried out, if the frequency f_j is used. Then $S_j = \{t \in T: t \text{ is a multiple of } t_j\}$ and $K = \cup_{j \in J} S_j$. The first shipment, for each frequency f_j , is carried out in 0 (Table 1).

In the following, we give some definitions and results, which will be used to formulate the model.

Definition 1. The instantaneous value of the inventory for product i in node A is equal to the initial value of the inventory at the node minus the cumulated quantity sent to node B up to time t plus the quantity produced:

$$I_{it}^A = d_i^A - \sum_{k \in K_t} s_{ik} + q_i \cdot t. \quad (1)$$

Definition 2. The instantaneous value of the inventory for product i in node B is equal to the initial value of the inventory at the node plus the cumulated quantity received by node A up to time t minus the quantity demanded:

$$I_{it}^B = d_i^B + \sum_{k \in K_t} s_{ik} - q_i \cdot t. \quad (2)$$

The following result holds:

Proposition 1. The average inventories in the nodes A and B are respectively equal to:

$$\bar{I}_i^A = d_i^A + q_i \cdot \frac{H}{2} - \frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it},$$

$$\bar{I}_i^B = d_i^B - \frac{q_i \cdot H}{2} + \frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it}.$$

Proof. See Appendix A.1.

As previously mentioned, our objective is to determine a periodic shipping strategy which minimises transportation and inventory costs. To this end, we have derived the following expression which represents the sum of all the costs involved in our problem formulation.

Proposition 2. The total sum of transportation and inventory costs is given by:

$$TC := \sum_{i \in I} \sum_{j \in J} \frac{h_i^A + h_i^B}{2} t_j \cdot q_i \cdot x_{ij} + \sum_{j \in J} \frac{c}{t_j} \cdot y_j.$$

Proof. See Appendix A.2.

Now, the model to minimize total inventory and transportation costs with perishable goods can be formulated as follows:

$$\min \left\{ \sum_{i \in I} \sum_{j \in J} \frac{h_i^A + h_i^B}{2} \cdot q_i \cdot t_j \cdot x_{ij} + \sum_{j \in J} \frac{c}{t_j} \cdot y_j \right\},$$

$$\sum_{j \in J} x_{ij} = 1, \quad i \in I, \quad (\text{c1})$$

$$t_j \cdot \sum_{i \in I} v_i \cdot q_i \cdot x_{ij} \leq r \cdot y_j, \quad j \in J, \quad (\text{c2})$$

$$\sum_{j \in J} t_j \cdot x_{ij} \leq ts_i, \quad i \in I, \quad (\text{c3})$$

$$x_{ij} \geq 0, \quad i \in I, \quad j \in J, \quad (\text{c4})$$

$$y - j \geq 0, \text{ integer} \quad j \in J \quad (\text{c5})$$

The objective function establishes the minimisation of total inventory and the transportation costs per time unit.

The set of constraints (c1) is the set of the demand constraints: for each product $i \in I$ the total quantity shipped in a cycle has to be equal to the quantity produced in node A and absorbed (sold) in node B .

The constraints (c2) are the capacity constraints: under the assumption that only the products shipped with the same frequency may share the same vehicle, the number of vehicles y_j used with frequency f_j is to be sufficient to load the whole quantity shipped at that frequency.

The constraints (c3) are the perishability constraints. Since we are considering perishable products, it is necessary that production, transportation and sale occur within the corresponding expiry date ts_i (for each product $i \in I$).

Since products are assumed to be processed in each node following a FIFO, the perishability constraints to be satisfied are:

$$I_{it}^A + I_{it}^B \leq q_i \cdot ts_i \quad i \in I, \quad t \in T. \quad (3a)$$

In this way, the total amount of product i , available in the system at every time $t \in T$, is to be less than or equal to the value $q_i \cdot ts_i$. It should be noted that, given a demand which is constant over time and equal to the production rate q_i and given the FIFO policy in handling products, the oldest product dates back at most ts_i instants of time. This product will be the first one sold in node B . It can be observed that the total inventory of each product in the two nodes is constant over time (for a detailed proof of this result in the multistage case see Bertazzi & Speranza, 1999a), i.e.:

$$I_{it}^A + I_{it}^B = d_i^A + d_i^B.$$

Then constraints (3a) become:

$$d_i^A + d_i^B \leq q_i \cdot ts_i \quad i \in I, \quad t \in T. \quad (3b)$$

Non stock-out constraints have been omitted from problem formulation due to the following observations:

- when formulating the objective function, the initial inventory levels have been considered as positive values (see Appendix A.1). Consequently, in the optimal solution, the initial inventory levels will be the smallest values which satisfy the whole set of constraints (non stock-out and perishability);
- as shown in Bertazzi and Speranza (1999b), the smallest value of the initial inventory for each product i , $i \in I$, which satisfies the non stock-out constraints in the two nodes is:

$$d_i^{A*} = \sum_{j \in J} t_j \cdot q_i \cdot x_{ij}, \quad i \in I,$$

$$d_i^{B*} = 0, \quad i \in I.$$

These values of the initial inventory have been used both in the objective function and in constraint (3b), thus allowing the stock-out constraints to be eliminated from the model. Therefore, the (3b) constraints can be rewritten as $\sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \leq q_i \cdot ts_i$. More precisely:

$$\sum_{j \in J} t_j \cdot x_{ij} \leq ts_i \quad \forall i \in I. \tag{3c}$$

It is to be underlined that problem feasibility requires that there be at least one frequency, whose shipment period is no longer than the earliest due date for the different products, i.e. $\exists j \in J: t_j \leq \min\{ts_i, i \in I\}$.

It is interesting to observe that the constraints on the expiry dates can be interpreted both as constraints on system inventory (see constraints (3b)) as well as constraints on the shipment frequencies for each product (see constraints (3c)).

By omitting constraints (3) and setting $h_i = \frac{h_i^A + h_i^B}{2}$, the present model comes back to the one presented in Speranza and Ukovich (1994), further studied by Bertazzi et al. (2000) and Speranza and Ukovich (1996).

3. Heuristic algorithms

In this section we introduce some heuristic algorithms which take perishable products into account. The starting points for these heuristics are the procedures presented in Bertazzi et al. (2000) opportunely modified to consider perishability. Two different types of heuristics have been analysed:

1. heuristics based on a limitation of the set of frequencies: first the number of vehicles requested (y_j) is determined for each frequency and then the $x_{i,j}$ are determined;
2. heuristics not based on the property of limitation of the set of frequencies: they simultaneously determine both the y_j 's and the x_{ij} 's.

The first group of heuristics can be further subdivided into two groups:

- 1.1 the heuristics which use the solution of the continuous time relaxation of the problem: these are based on the simple idea of using one single shipment frequency, corresponding to a single optimal period t^* for all the products, as obtained solving the problem by relaxing the set of the permitted shipping frequencies to continuous time;
- 1.2 heuristics which are not based on the continuous time relaxation: they are based on the idea of using two or more shipping frequencies.

Fig. 1 provides the classification of all the proposed heuristics.

3.1. Heuristic based on the frequencies set limitation

In this class, we propose four different heuristics (Full, Best, Eoq-s, Eoq-l), which are suitable modifications of the heuristic proposed by Bertazzi et al. (2000). Each heuristic is based on the iterative application of two

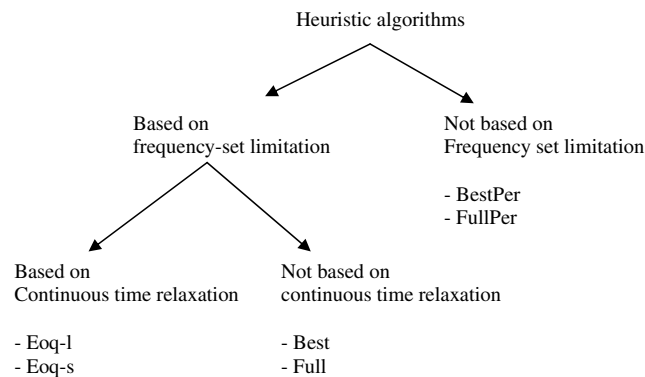


Fig. 1. Classification of the proposed heuristics.

steps: the first one is specific for each heuristic and aims to determine the number of vehicles y_j for each frequency f_j . The second step is common to all the heuristics and determines the optimal matrix x_{ij} given the particular vector y_j as input. After the y_j calculation and the corresponding x_{ij} , the value of the cost function is estimated. The process is iterated for different y_j vectors, among all the generated pairs (y_j, x_{ij}) . The one with the smallest total cost will be chosen. Each y_j vector can be separately computed, with respect to the associated matrix x_{ij} , thanks to the following property, which limits the set of frequencies.

In the formulation of the model, we have assumed that a known set of frequencies, i.e. $J = \{1, 2, \dots, |J|\}$, is given. Moreover, the period t_j between two consecutive shipments is assumed to be an integer number. In the remainder, we will assume that frequencies are organised in a decreasing order: i.e. f_1 is the highest frequency available or, in other words, the frequency with the smallest period $t_1 = 1/f_1$ between two consecutive shipments.

The constraints on perishability are important when ascribing the products to the transportation frequencies. These constraints make the determination of vector y_i difficult, independently of the quantities x_{ij} .

As a result of an opportune limitation of the frequency set, it will be shown how it is possible to separate the determination of the y_j vector from that of x_{ij} , generating feasible solutions.

Given the initial set of frequencies J , we arrange a new set of frequencies J' which contains all the frequencies of J with a shipping period shorter than the earliest expiry dates of all the products, that is:

$$J' = \{j \in J : t_j \leq \min \{ts_i, i \in I\}\}. \tag{4}$$

Proposition 3. *For each attribution of the products to the frequencies in J' (i.e. for any value of the x_{ij} , $i \in I$ and $j \in J'$), the constraints on due dates are respected.*

Proof. We have to show that $\forall x_{ij}, i \in I$ and $j \in J'$, we have $\sum_{j \in J'} t_j x_{ij} \leq ts_i$

In fact, it is possible to write that:

$$\sum_{j \in J'} t_j x_{ij} \leq \sum_{j \in J'} \min \{ts_i, i \in I\} x_{ij} = \min \{ts_i, i \in I\} \sum_{j \in J'} x_{ij} = \min \{ts_i, i \in I\} \leq ts_i. \quad \square \tag{5}$$

We can conclude that for any allocation of the products to the frequencies in J' , the products are always produced, shipped and sold by the corresponding due dates.

In the following heuristic algorithms, the above frequency set limitation is applied indirectly, by imposing that the number of vehicles used with frequencies determining a period larger than $\min \{ts_i, i \in I\}$ is null. This fact implies that $x_{ij} = 0 \forall j: t_j > \min \{ts_i, i \in I\}$, and **Proposition 3** guarantees that all the possible assignments x_{ij} $i \in I$ and $j \in J'$ are feasible, with respect to perishability constraints. As a consequence of the modification of set J , the time horizon is now equal to $H = \text{mcm}\{t_j, j \in J'\}$, i.e. a value which is generally larger than $\min \{ts_i, i \in I\}$.

It is important to observe that the use of a limited set has important practical implications on the Supply Chain management, as the production system does not have to identify and deal separately with the products, according to their expiry dates. On the contrary, if the productive system managed the products differently, according to the different expiry periods, high operational costs would be incurred.

So as to simply identify the solution, the present Section offers a relaxation of the problem, considering continuous time shipping frequencies (see Section 2 for the discrete case). More precisely, a single optimal period t^* may be determined for all products and this can be computed as follows:

$$t^* = \min \left\{ \sqrt{\frac{c}{\sum_{i \in I} \frac{h_i^A + h_i^B}{2} q_i}}, \frac{r}{\sum_{i \in I} v_i q_i}, \min \{ts_i, i \in I\} \right\}. \tag{6}$$

It is easy to note that the first of the three quantities is the classic “Wilson’s formula” for the economic order quantity (see [Erlenkotter, 1989](#)). The second takes into account the finite capacity of the vehicles, while the last one introduces the product perishability. It is also plain that t^* does not necessarily correspond to one of the frequencies belonging to J . Therefore, the following heuristics use formula (6) to determine a feasible solution for the initial problem.

3.1.1. Eoq-l

1. Given the set of frequencies J and the dates of expiry ts_i for each product i , compute set J' introducing the limitation of the set J .
2. Round period t^* (solution of (6)) to the nearest available period in J' , larger than or equal to t^* , and indicate such period as $t_k = \min_{j \in J'} \{t_j : t_j \geq t^*\}$.
If no period satisfies this condition, set $t_k = t_{|J'|}$.
3. Compute the volume per unit of time to transport (W), the largest number of full vehicles at the frequency f_k (i.e. y_k) and update the remaining volume that is to be sent:

$$W = \frac{\sum_{i \in I} q_i H v_i}{H} = \sum_{i \in I} q_i v_i, \quad y_k = \left\lfloor \frac{WH}{H/t_k} \cdot \frac{1}{r} \right\rfloor = \left\lfloor \frac{t_k W}{r} \right\rfloor, \quad W = W - \frac{y_k \cdot r}{t_k}. \quad (7)$$

If $W > 0$, since it is not possible to saturate vehicles with a frequency greater than or equal to f_k , send the remaining volume to the smallest frequency available in J' :

$$y_k = \left\lfloor \frac{t_{|J'|} W}{r} \right\rfloor. \quad (8)$$

4. Apply the algorithm of Frequency Assignment (1), described in Section 3.1.5.
5. Compute the total cost for time unit.

3.1.2. Eoq-s

1. Given the set of frequencies J and the expiry dates ts_i for each product i , compute set J' introducing the limitation of the set J .
2. Round off the period t^* (solution of (6)) to the nearest available period in J' smaller than or equal to t^* , indicate such period as $t_k = \max_{j \in J'} \{t_j : t_j \leq t^*\}$. If no periods in J' satisfy this condition, set $t_k = t_1$.
3. Compute the volume for unit of time to transport (W), and the number of vehicles necessary to transport all the products at frequency f_k (i.e. y_k) and update the remaining volume that has to be sent:

$$W = \sum_{i \in I} q_i v_i; \quad y_k = \left\lfloor \frac{t_k \cdot W}{r} \right\rfloor.$$

4. Apply the algorithm of Frequencies Assignment (1), described in Section 3.1.5.
5. Compute the total cost per time unit.

3.1.3. Full

This heuristic is based on the idea of carrying out a shipment as soon as a vehicle is filled up.

1. Given the set of frequencies J and the expiry dates ts_i for each product i , compute set J' carrying out the limitation of the set J .
2. Compute the volume per unit of time to transport $W = \sum_{i \in I} q_i v_i$.
3. Determine one or more feasible y_j vectors as follows:
 - 3.1. starting from the highest frequency (f_1) and going all the way to the lowest ($f_{|J'|}$), calculate the full-load vehicles for each frequency and update the volume per unit of time (W) remaining to transport, i.e. for j from 1 to $|J'|$ compute:

$$y_j = \left\lfloor \frac{t_j \cdot W}{r} \right\rfloor; \quad W = W - \frac{y_j \cdot r}{t_j}.$$

- 3.2. If $W > 0$, introduce new vehicles to transport all the volume produced in H . Then, determine $|J'|$ admissible vector y_j , repeating steps 3.2.1 and 3.2.2 with j starting from 1 to $|J'|$.
 - 3.2.1 From point 3.1 vector y_j , introduce a new vehicle at the frequency j , i.e. $y_j = y_j + 1$.

3.2.2 Update the number of vehicles necessary at the frequencies lower than f_j . The introduction of an additional vehicle to the frequency f_j allows the transportation of all the volume of products not assigned to the larger frequencies and makes it useless to use frequencies inferior to f_j .

4. For each computed vector y_j , apply the algorithm of Frequencies Assignment (1), described in Section 3.1.5.
5. For each solution (y_j, x_{ij}) , compute the value of the total cost per time unit and choose the solution with the lowest cost.

3.1.4. Best

This heuristic is based on the limitation of the set of frequencies using only two frequencies in set J' . In particular, we assume the highest frequency (f_1) is always used and the heuristic itself establishes which other frequency could be used, if any, to diminish the total costs.

1. Given the set of frequencies J and the expiry dates ts_i for each product i , compute set J' introducing the limitation of set J .
2. Compute the volume per unit of time to transport $W = \sum_{i \in I} q_i v_i$.
3. Determine y_j vectors, containing two shipment frequencies, on the basis of points 3.1 and 3.2.

3.1. Calculate the number y_1 of fully-loaded vehicles that it is possible to send at the highest frequency, and update the volume per time unit remaining:

$$y_1 = \left\lfloor \frac{t_1 W}{r} \right\rfloor; \quad W = W - \frac{y_1 \cdot r}{t_1}.$$

3.2. If $W > 0$, for each frequency j belonging to J' , compute the number of vehicles necessary, at the j th frequency, to ship all the volume not assigned to y_1 at point 3.1.

Put : $y_j = y_j + 1$ if $j = 1$,

$$y_j = \left\lfloor \frac{t_j W}{r} \right\rfloor, \quad \text{otherwise.}$$

4. For each computed vector y_j , apply the algorithm of Frequencies Assignment (1), described in Section 3.1.5.
5. For each vector y_j , compute the value of the total cost per time unit and choose the solution with the lowest cost.

3.1.5. Frequencies Assignment (1)

In this paragraph, the algorithm used for the allocation of the products to the shipment frequencies is assigned. This algorithm is common to all the heuristics based on the frequencies set limitation. In Speranza and Ukovich (1996) it is demonstrated that, given the number of vehicles used at each frequency (y_j), its application determines optimal assignments of the products to the frequencies.

In order to apply this algorithm, it is necessary to arrange the products in a non-increasing order of the value: $\frac{h_i}{v_i} = \frac{h_i^a + h_i^b}{2v_i}$ and the set J' in increasing order of period t_j . The proposed algorithm aims to load the vehicles y_j starting from the products with the highest \bar{h}_i/v_i ratio.

The algorithm can be described as follows:

1. Take the first element i' of I and j' of J'
2. If it is possible to transport all the products i' with the frequency j' , i.e. the following inequality is satisfied:

$$v_{i'} q_{i'} - \sum_{j \in J} v_{i'} q_{i'} x_{i'j} \leq r \frac{y_{j'}}{t_{j'}} - \sum_{i \in I} v_i q_i x_{ij'}$$
 then

- 2.1. transport all products i' with frequency j' : $x_{i'j'} = 1 - \sum_{j \in J} x_{i'j}$ and delete the product i' from the set I .
- 2.2. vice versa, transport with frequency j' only the fraction of products i' saturating the remaining vehicles:

$$x_{i'j'} = \frac{r}{v_{i'} q_{i'} t_{j'}} y_{j'} - \sum_{i \in I} \frac{v_i q_i}{v_{i'} q_{i'}} x_{ij'}$$
 and delete the frequency j' from set J' .

3. Repeat step 1–2 until $I = \emptyset$.

After carrying out this algorithm, we have assigned a value to vector y_j and to the matrix x_{ij} and it is possible to calculate the total cost per time unit corresponding to such a solution.

3.2. Heuristic not based on the frequencies set limitation

It is important to observe that if the property of frequencies set limitation is not used, this implies two important consequences:

1. it is not possible to separate the computations of y_j and x_{ij} ;
2. a different way needs to be identified to guarantee that the solution obtained satisfies the perishability constraints.

The heuristics are based on the assumption that no fraction of any product (x_{ij}) can be shipped in periods longer than the expiry date of the product.

3.2.1. FullPer

The basic hypothesis is related to a frequent and real situation, i.e. a vehicle must be sent as soon as the supplier has manufactured a volume of the product sufficient to saturate the vehicle itself.

1. Determine the smallest frequency (f_{\min}) as follows:
 - 1.1 Compute the volume per time unit to send $W = \sum_{i \in I} q_i v_i$.
 - 1.2 Starting from the highest frequency (f_1) until the assignment of all products to a frequency, repeat the following steps 1.2.1–1.2.4.
 - 1.2.1. Calculate the maximum number of full vehicles that it is possible to ship at frequency j

$$y_{\text{sat},j} = \left\lfloor \frac{t_j W}{r} \right\rfloor$$

- 1.2.2. Calculate the minimum number of vehicles that is necessary to ship at frequency j ($y_{\text{nec},j}$), giving priority to the products expiring within the period $t_j = 1/f_j$ and which are not yet assigned to any frequency ($\sum_{j \in J} x_{ij} < 1$).

$$W' = \frac{\sum_{i \in I'} q_i H v_i}{H} = \sum_{i \in I'} q_i v_i, \quad \text{where } I' = \left\{ i \in I : \sum_{j \in J} x_{ij} < 1 \text{ and } t_{s_i} = t_j \right\},$$

$$y_{\text{nec},j} = \left\lceil \frac{t_j W'}{r} \right\rceil.$$

- 1.2.3. Choose the maximum (y_j) between $y_{\text{sat},j}$ and $y_{\text{nec},j}$ and update the volume per time unit to be shipped:

$$y_j = \max \{y_{\text{sat},j}, y_{\text{nec},j}\}, \quad W = W - \frac{y_j \cdot r}{t_j}.$$

- 1.2.4. For each y_j , apply the algorithm of Frequencies Assignment (2), described in Section 3.2.3.
2. For j from f_1 to f_{\min} , iterate steps 1.1 and 1.2, by increasing by one unit (+1) per iteration the number of vehicles that travel at the frequency j th. At the end of the iterations, f_{\min} vectors y_j are obtained with the respective matrices x_{ij} associated.
3. For each solution (y_j, x_{ij}), compute the value of the total cost per time unit and choose the solution with the lowest cost.

3.2.2. BestPer

The last heuristic is the Best performing one, as it allows the utilisation of only two shipment frequencies with periods longer than the minimum expiry date of the products. The aim is to determine whether the restric-

tion introduced (two frequencies) involves a significant deterioration of the objective function with respect to the optimal solution.

1. Choose the first shipment frequency as the highest available (f_1) and calculate the volume per time unit to be sent (W) and the maximum number of saturated vehicles ($y_{sat,1}$) that it is possible to send at such a frequency:

$$W = \sum_{i \in I} q_i v_i, \quad y_{sat,1} = \left\lceil \frac{t_1 W}{r} \right\rceil.$$

2. Going from the highest shipment frequency ($j = 1$) all the way to the lowest one ($j = |J|$), iterate from 2.1 to 2.5 points for j times.

- 2.1. Calculate the minimum number of vehicles to send at the highest frequency (f_1), the j th frequency being the second rate to be used.

$$W' = \frac{\sum_{i \in I'} q_i H v_i}{H} = \sum_{i \in I'} q_i v_i, \quad \text{where } I' = \{i \in I : ts_i < t_j\},$$

$$y_{nec,1} = \left\lceil \frac{t_1 W'}{r} \right\rceil.$$

- 2.2. Choose the maximum (y_1) between $y_{sat,1}$ and $y_{nec,1}$ and update the volume per time unit that must be sent:

$$y_1 = \max \{y_{sat,1}, y_{nec,1}\}, \quad W = W - \frac{y_1 \cdot r}{t_1}.$$

- 2.3. For each y_j , apply the algorithm of Frequencies Assignment (2), described in Section 3.2.3.

- 2.4. Calculate the number of vehicles necessary to send the remaining volume at the frequency f_j .

$$y_1 = y_1 + \left\lceil \frac{t_1 W}{r} \right\rceil \quad \text{for } j = 1.$$

$$y_j = \left\lceil \frac{t_1 W}{r} \right\rceil \quad \text{otherwise.}$$

3. For each solution (y_j, x_{ij}), compute the value of the total cost per time unit and choose the solution with the lowest cost.

3.2.3. Frequencies Assignment (2)

This algorithm acts using the set of products I and a particular frequency f_j . It is necessary to arrange the products in a decreasing order of \bar{h}_i/v_i ratio. The same considerations made in the algorithm of the previous assignment are still valid. Alternatively, a priority allocation is introduced to frequency f_j for those products which are not yet allocated, with an expiry period equal to or shorter than t_j .

The algorithm can be presented as follows:

1. Arrange set I'' including all the products not completely assigned to one frequency and with expiry date equal to or less than t_j :

$$I'' = \{i \in I : ts_i \leq t_j\}.$$

2. Arrange set I' , whose elements are the products belonging to I but not included in I'' ; order I' products in a decreasing order according to \bar{h}_i/v_i ratio.

3. Send the products pertaining to I'' set, together with their fractions which are not yet assigned to a frequency, with the frequency f_j . Extract the product from set I :

$$x_{i''j} = 1 - \sum_{j \in J} x_{i''j}, \quad \text{where } i'' \in I''.$$

4. Consider the first element i' of I' . If it is possible to send all product i' with frequency j , the following expression holds:

$$v_{i'}q_{i'} - \sum_{j \in J} v_{i'}q_{i'}x_{i'j} \leq r \frac{y_j}{t_j} - \sum_{i \in I} v_i q_i x_{ij},$$

and therefore

- 4.1. send all i' at frequency j , setting $x_{i'j} = 1 - \sum_{j \in J} x_{i'j}$ and delete the product i' from sets I and I' :
 $I = I \setminus \{i'\}$, $I' = I' \setminus \{i'\}$. Vice versa.
- 4.2. Send with frequency j only that fraction of i' that saturates the residual vehicles:

$$x_{i'j} = \frac{r}{v_{i'}q_{i'}t_j} y_j - \sum_{i \in I} \frac{v_i q_i}{v_{i'}q_{i'}} x_{ij}.$$

5. Iterate points 4 up to the complete saturation of the vehicles travelling at frequency f_j .

4. Experimental analysis

In the previous paragraphs, six heuristics have been introduced. They can be used to solve the problem of cost minimisation in a two-node Supply Chain dealing with perishable products. Here below, some experimental results are presented to compare their performance. The optimal solution for each generated instance has been calculated using the software CPLEX 6.6. The mechanism adopted for case generation is synthesised in Fig. 2.

Tests are divided into two classes, according to the two different values that parameter α , $0 < \alpha < 1$, may have. Notation t_{\max} and t_{\min} will refer to the maximum and the minimum period, respectively, between two consecutive shipments belonging to set J (i.e. $t_{\max} = \max\{t_j, j \in J\}$ and $t_{\min} = \min\{t_j, j \in J\}$). The expiry dates for the products have been generated according to a uniform distribution in the interval $[\alpha t_{\max}, t_{\max}]$. For the first class of experiments, we choose $\alpha = 0.6$, while for the second one $\alpha = t_{\min}/t_{\max}$. The value $\alpha = 0.6$ refers to a situation in which expiry dates are concentrated between 60% and 100% of t_{\max} . Consequently, when assigning the products to the chosen frequencies (property of frequencies set limitation), we can plan activities only on the basis of frequencies higher than $[0.6t_{\max}]$. The category with $\alpha = t_{\min}/t_{\max}$ describes a situation in which expiry dates range between t_{\min} and t_{\max} .

Each class of experiments consists of 180 instances characterised by different values of the pair $(|J|, |I|)$, with $|J|$ cardinality of the frequencies available set and $|I|$ cardinality of the products set. Three different values were considered for $|J|$ (i.e., 5, 10 and 15) and for $|I|$ (i.e., 100, 500 and 1000). We have considered all the possible pairs of $(|J|, |I|)$ thus generating nine groups, each consisting of 20 different instances.

Each group has been divided into subgroups of instances characterised by different values of the pair (Q, M) , with Q and H , respectively, the sets of sampling values for the production rate (or rate of demand) and for the holding costs in the first node. The production rate has been generated according to a uniform distribution in one of the two following intervals: $Q = [10, 100]$ and $Q = [100, 1000]$. The holding cost in the first node has been generated according to a uniform distribution: $M = [0.001, 0.006]$ and $M = [0.006, 1]$. The holding cost in the second node is obtained by increasing the cost in the first node by a percentage sampled from the uniform distribution $[1, 1.25]$. In order to test all the possible combinations of (Q, M) pairs, each group was divided into four subgroups consisting of five instances each.

Finally, for all the instances tested, we have generated the unitary volume of each product i , $i \in I$ sampling it from a uniform distribution $[0.0005, 0.001]$. Without loss of generality, the maximum volume for each vehicle was assumed to be equal to 1, i.e. each truck can load from 1000 to 2000 units of a single product.

The transportation cost per journey performed is set at 300.

In order to measure the performance of the heuristics, the percentage error was adopted:

$$\text{error \%} = \frac{\text{heuristic solution} - \text{optimal solution}}{\text{optimal solution}} \times 100$$

The average errors are shown in the following table.

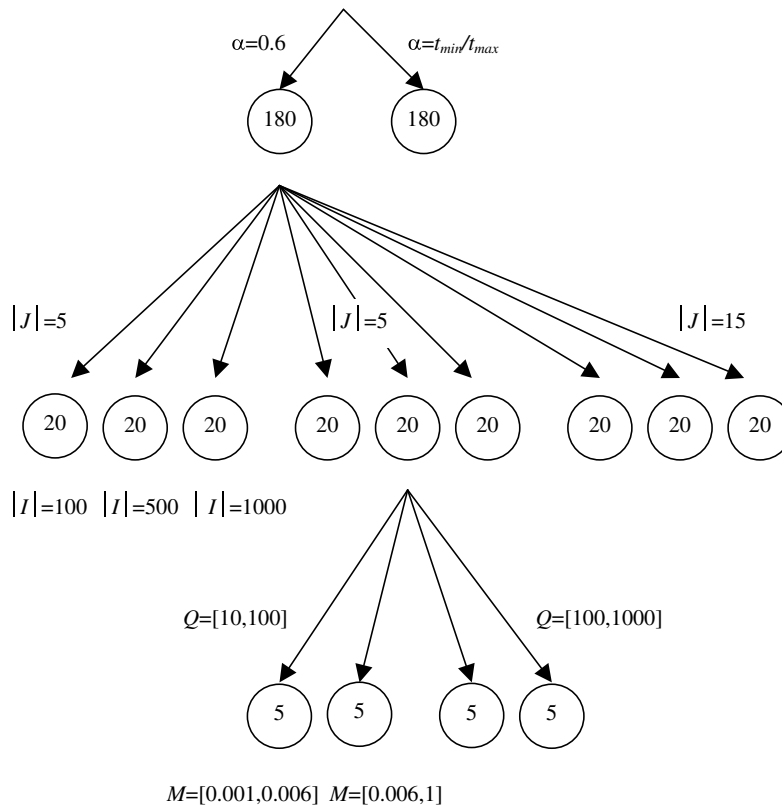


Fig. 2. Organisation of the cases for experimental analysis.

In Table 2, the average errors for all the heuristics in the 360 instances are shown. The best heuristics are those which are not based on the limitation of the set of frequencies (i.e. FullPer and BestPer), while the performance of the heuristics based on the limitation of the frequency set are not as good as expected, i.e. Eoq-s, Eoq-l, Best and Full. To provide a more detailed analysis, Tables 3 and 4 show the maximum, the minimum and average percentage errors found when $\alpha = 0.6$ and $\alpha = t_{\min}/t_{\max}$.

For $\alpha = 0.6$, all the heuristics have a good performance, with the exception of heuristic Eoq-s. When $\alpha = t_{\min}/t_{\max}$, the errors increase for all the heuristics and BestPer remains the algorithm that offers the best results.

5. Conclusion

The present paper addresses the problem of minimizing the transportation and inventory costs associated with the shipment of various perishable products from a single origin to a single destination. A mixed-integer programming model is proposed and six heuristic solution algorithms are presented. The heuristics have been tested and compared for several instances. The experimental results show that BestPer heuristic provides the best performance of all the classes of instances.

Appendix A

A.1. Proof of Proposition 1

Starting with node *B*, let us consider the generic outline over time for the instantaneous inventory (see Fig. 3) at node *B* (the buyer).

Table 2
Average percentage error

| Heuristic | Full | Best | Eqq-l | Eqq-s | FullPer | BestPer |
|-----------|-------|-------|-------|-------|---------|---------|
| error % | 1.029 | 1.018 | 1.101 | 1.932 | 0.310 | 0.123 |

Table 3
Max, min and average error % for $\alpha = 0.6$

| Heuristic | Full | Best | Eqq-l | Eqq-s | FullPer | BestPer |
|-----------------|-------|-------|-------|-------|---------|---------|
| Max error % | 6.419 | 4.824 | 4.824 | 23.92 | 6.419 | 2.178 |
| Min error % | 0 | 0 | 0.018 | 0.086 | 0 | 0 |
| Average error % | 0.273 | 0.251 | 0.417 | 2.080 | 0.236 | 0.111 |

Table 4
Max, min and average error % for $\alpha = t_{\min}/t_{\max}$

| Heuristic | Full | Best | Eqq-l | Eqq-s | FullPer | BestPer |
|-----------------|--------|--------|--------|--------|---------|---------|
| Max error % | 21.170 | 21.170 | 21.170 | 21.170 | 7.962 | 4.145 |
| Min error % | 0.041 | 0.041 | 0.041 | 0.041 | 0 | 0 |
| Average error % | 1.785 | 1.785 | 1.785 | 1.785 | 0.384 | 0.135 |

The step-wise shape (dashed lines in Fig. 3 profile) overestimates the profile of the instantaneous inventory. The average area under the profile can be easily computed:

$$\bar{I}_i^B = \frac{1}{H} \cdot \sum_{t \in T} I_{it}^B.$$

At each time instant, the area of the triangle (dashed lines) overestimating the real value of the inventory is equal to $q_i/2$ and, consequently, the area under the profile is:

$$\bar{I}_i^B = \frac{1}{H} \cdot \sum_{t \in T} \left(I_{it}^B - \frac{q_i}{2} \right). \tag{A.1}$$

Replacing the value of I_{it}^B (see (2)) in (A.1) expression, and observing that $\sum_{t \in T} t = \frac{H(H-1)}{2}$, we get:

$$\begin{aligned} \bar{I}_i^B &= \frac{1}{H} \cdot \sum_{t \in T} \left(d_i^B + \sum_{k \in K_t} s_{ik} - q_i \cdot t - \frac{q_i}{2} \right) = \frac{1}{H} \cdot \sum_{t \in T} d_i^B + \frac{1}{H} \sum_{t \in T} \sum_{k \in K_t} s_{ik} - \frac{1}{H} \sum_{t \in T} q_i \cdot t - \frac{1}{H} \sum_{t \in T} \frac{q_i}{2} \\ &= d_i^B + \frac{1}{H} \cdot \sum_{t \in T} \sum_{k \in K_t} s_{ik} - q_i \cdot \frac{H-1}{2} - \frac{q_i}{2} = d_i^B - q_i \cdot \frac{H}{2} + \frac{1}{H} \sum_{t \in T} \sum_{k \in K_t} s_{ik}. \end{aligned}$$

Moreover, the following expression is valid:

$$\sum_{t \in T} \sum_{k \in K_t} s_{ik} = \sum_{i \in K} (H-t) \cdot s_{it}.$$

In fact, let us consider a shipment that occurs at time k' . The term $s_{ik'}$ will be counted $H-k'$ times. Therefore \bar{I}_i^B can be calculated as follows:

$$\bar{I}_i^B = d_i^B - q_i \frac{H}{2} + \frac{1}{H} \sum_{i \in K} (H-t) \cdot s_{it}. \tag{A.2}$$

Since

$$\bar{I}_i^A = \bar{I}_i - \bar{I}_i^B = (d_i^A + d_i^B) - \bar{I}_i^B \tag{A.3}$$

then replacing (A.2) in (A.3) we immediately get:

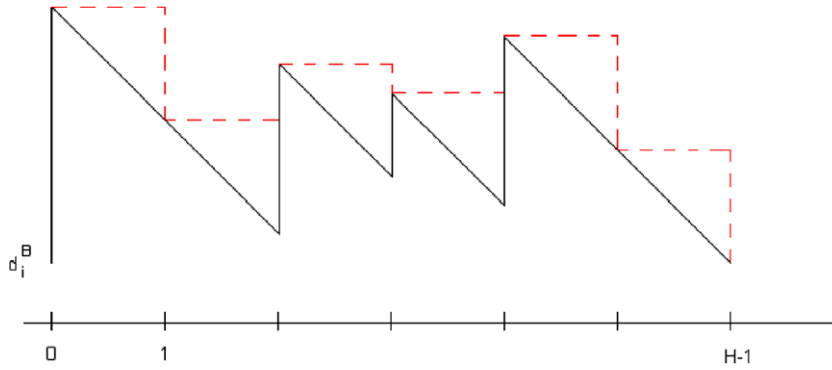


Fig. 3. Generic trend of the instantaneous inventory in node B.

$$\bar{I}_i^A = d_i^A + q_i \cdot \frac{H}{2} - \frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it}. \quad \square$$

A.2. Proof of Proposition 2

Let's indicate as HC_i the average inventory cost for product i :

$$\begin{aligned} HC_i &= h_i^A \cdot \bar{I}_i^A + h_i^B \cdot \bar{I}_i^B \\ &= h_i^A \cdot \left(d_i^A + q_i \cdot \frac{H}{2} - \frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it} \right) + h_i^B \cdot \left(d_i^B - q_i \cdot \frac{H}{2} + \frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it} \right) \\ &= h_i^A \cdot d_i^A + h_i^B \cdot d_i^B + (h_i^B - h_i^A) \cdot \left(\frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it} - \frac{q_i}{2} \cdot H \right). \end{aligned}$$

In the optimal solution, d_i^A and d_i^B values will satisfy the constraint of non stock-out (see Bertazzi & Speranza, 1999a, for a proof), then:

$$\begin{aligned} d_i^{A*} &= \sum_{j \in J} t_j \cdot q_i \cdot x_{ij}, \\ d_i^{B*} &= 0. \end{aligned}$$

Replacing the two last expressions in HC_i :

$$\begin{aligned} HC_i &= h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} + (h_i^B - h_i^A) \cdot \left(\frac{1}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it} - \frac{q_i \cdot H}{2} \right) \\ &= h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} + \frac{h_i^B - h_i^A}{H} \cdot \sum_{t \in K} (H - t) \cdot s_{it} - (h_i^B - h_i^A) \cdot \frac{q_i \cdot H}{2}. \end{aligned} \quad (A.4)$$

By observing that $\sum_{t \in K} s_{it} = q_i \cdot H$, we have:

$$\sum_{t \in K} (H - t) \cdot s_{it} = \sum_{t \in K} H \cdot s_{it} - \sum_{t \in K} t \cdot s_{it} = q_i \cdot H^2 - \sum_{t \in K} t \cdot s_{it}. \quad (A.5)$$

Substituting (A.5) in (A.4) it follows:

$$HC_i = h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} - \frac{h_i^B - h_i^A}{H} \cdot \sum_{t \in K} t \cdot s_{it} + (h_i^B - h_i^A) \cdot \frac{q_i \cdot H}{2}.$$

By the definition of s_{it} , $\forall i, t$, we get:

$$\sum_{i \in K} t \cdot s_{it} = \sum_{i \in K} t \cdot \sum_{j \in J_i} t_j \cdot q_i \cdot x_{ij} = \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} (H - n \cdot t_j).$$

In fact, each shipment ($t_j \cdot q_i \cdot x_{ij}$) occurs $|S_j| = H/t_j$ times. Each shipment is weighted with a parameter that depends on the instant in which the shipment occurs (first shipment at $n = 1$ and last shipment at $n = H/t_j$). Thus,

$$\begin{aligned} \sum_{i \in K} t \cdot s_{it} &= \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} (H - n \cdot t_j) = \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} H - \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} n \cdot t_j \\ &= \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \frac{H^2}{t_j} - \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} n \cdot t_j = H^2 \cdot q_i \cdot \sum_{j \in J} x_{ij} - \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} n \cdot t_j. \end{aligned} \quad (\text{A.6})$$

Given that $\sum_{j \in J} x_{ij} = 1$, the last term can immediately be rewritten as follows: $H^2 \cdot q_i \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \sum_{n=1}^{H/t_j} n \cdot t_j$.

The last term can be rewritten as follows:

$$\sum_{n=1}^{H/t_j} n \cdot t_j = t_j \cdot \sum_{n=1}^{H/t_j} n = \frac{t_j}{2} \cdot \frac{H}{t_j} \cdot \left(\frac{H}{t_j} + 1 \right) = \frac{H}{2} \cdot \left(\frac{H}{t_j} + 1 \right),$$

and then:

$$\sum_{i \in K} t \cdot s_{it} = H^2 \cdot q_i - \frac{H}{2} \cdot \sum_{j \in J} \left(\frac{H}{t_j} + 1 \right) t_j \cdot q_i \cdot x_{ij}.$$

Rewriting the holding cost for the product i on the basis of the previous result:

$$\begin{aligned} HC_i &= h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} + \frac{h_i^B - h_i^A}{2} \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} \cdot \left(\frac{H}{t_j} + 1 \right) - (h_i^B - h_i^A) \cdot \frac{q_i \cdot H}{2} \\ &= h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} + \frac{h_i^B - h_i^A}{2} \cdot H \cdot q_i \cdot \sum_{j \in J} x_{ij} + \frac{h_i^B - h_i^A}{2} \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} - (h_i^B - h_i^A) \cdot \frac{q_i \cdot H}{2} \\ &= h_i^A \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij} + \frac{h_i^B - h_i^A}{2} \cdot \sum_{j \in J} t_j \cdot q_i \cdot x_{ij}. \end{aligned}$$

By summing the inventory costs that determine the total transportation costs, we finally get:

$$\sum_{i \in I} HC_i + \sum_{j \in J} \frac{c}{t_j} \cdot y_j = \sum_{i \in I} \sum_{j \in J} \frac{h_i^A + h_i^B}{2} t_j \cdot q_i \cdot x_{ij} + \sum_{j \in J} \frac{c}{t_j} \cdot y_j \quad \square$$

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