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# The consignment stock of inventories: industrial case and performance analysis

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**Abstract**

The "Consignment Stock" technique is a novel approach to the management of inventories in supply chains. It is based on an improved collaboration between the company and its suppliers, one that is acquiring growing importance in industrial environments, as the authors have found in Italy. The main aim of the present work is to describe the technique itself, thus underlining its potential benefits and pitfalls. The case proposed refers to a company manufacturing components for the automotive industry. Essentially, the company offered its suppliers the opportunity of stocking part of the items in its own warehouses, with the agreement that they would guarantee over time an inventory level between a set minimum and a maximum value.



*Keywords:* Supply chain; Inventory management; Consignment stock

## 1. Introduction and consignment stock practice

There is no question that over the past decades an increasing amount of research has concerned the importance of establishing a profitable vertical relationship between companies and suppliers, especially in management literature. In particular, strong interaction and reliable collaboration between these two actors have emerged as strategic issues and powerful instruments for maintaining or acquiring competitive advantages in a dynamic and selective market. Indisputably, this issue also plays a pivotal role in inventory policies and management. Yet, despite the growing number of studies and theoretical models developed, the operational research still seems to be frequently divorced from industrial reality. Meanwhile, several practices that are not dealt with in the literature show up, acquire importance, and prove to be successful.

This is the case of consignment stock (CS) management of provisioning, which, regardless of some similarities with the common $(s, S)$ policy, reveals significant innovative contributions. Nowadays, this practice has been widely adopted in Italy, and is consistently gaining consensus among both small and large firms. Both business press and private sources have also confirmed recent CS applications between Italy and other countries. Under a CS policy, the relationship

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between a company and a supplier is based on the following simple rules:

1. The supplier will guarantee the company the continuity of an available stock between a minimum level $s$ and a maximum level $S$: the stock will be stored in the company's raw material depots, close to the production lines.
2. The company may draw on raw materials daily, according to its needs. The supplier is paid for these materials according to their agreement, hypothetically up to a daily frequency, so that the information concerning the consumption trend is also constantly refreshed and immediately transferred to the supplier.

In such a way, the continuous replenishment from the supplier protects the company against demand fluctuations and costs determined by eventual stockout may also be debited to the supplier, by means of contract penalties. On the other hand, the supplier has a better perception of his customer's requirements: lower stocking costs are incurred and the continuous evolution of market demand is directly perceived thanks to an electronic data interchange (EDI) interface. Several benefits are immediately evident:

1. The company always has raw material available.
2. The company pays for raw material consumption only when the items are drawn on for use.
3. The supplier saves holding costs and may organise his production in different ways, also with respect to eventual third parties.
4. A renewed and reinforced link is set up between the company and its supplier.

However, CS requires the accurate definition of various parameters, i.e. $s$ and $S$, and a constant attention to the information flow, that is, the electronic transmission to the supplier of item consumption. On this basis, the supplier can foresee the consequences of a better management of his own production, being freed from the bounds implicit in the strict EOQ practice (e.g. handling large but infrequent orders).

This paper seeks to provide a coherent framework within which to understand the success of CS. We ask why these changes are taking place. Is it a mere coincidence that various firms are adopting CS policies or is there a rationale behind their choice? If this is the case, how should their choice be implemented?

So as to grasp both the *why* and the *how* of CS, the rest of the paper is organised as follows. Section 2 briefly presents the main results of the literature, while Section 3, using the results of a computational simulation, shows how CS policy may outperform previous models. In other words, we provide a tentative proof that CS is a *rational* choice. Section 4 addresses the main tactical questions that follow the decision to adopt CS. This problem is tackled with the benefit of insights offered by a case study. Concluding remarks follow.

## 2. Literature review

One of the major outcomes of the inventory theory has been to show that $(s, S)$ policies are optimal for a class of dynamic inventory models with random periodic demands and fixed ordering costs. Under a common $(s, S)$ policy, if the inventory level at the beginning of a period is lower than the reorder point $s$, then a sufficient quantity must be ordered to achieve an inventory level $S$. Literature on the subject is ample and covers a wide time span. E.g., Iglehart (1963) obtained the stationary distribution of the inventory/backlog and developed an explicit formula for evaluating the stationary average cost, with the appropriate assumptions. Zheng (1991) has provided a rigorous proof of the optimality of an $(s, S)$ policy for the model of Veinott and Wagner (1965). Sethi and Cheng (1997) broadened some previously rigid assumptions in favour of more realism, and still demonstrated the optimality of $(s, S)$ policies dealing with a given distribution of demand.

In general, many practical inventory replenishment problems satisfy reasonably well the mathematical conditions under which this type of policy is convenient; however, complex analytical methods for computing the best (or even "a good") policy are rarely used, because, according to some practitioners, they are prohibitively expensive.

Several researches tried to show that this objection does not respond to reality. Generally speaking, inventory models have been discussed extensively in literature, whereas the treatment of interactions between buyers and suppliers has developed more recently (Goyal and Gupta, 1989). In 1977, Goyal (Goyal, 1977) suggested a joint economic lot size model where the objective is to minimise the total relevant costs for both the vendor and the buyer. The model was generalised by Banerjee (1986) and Goyal (1988) himself. While these later models assume that there's a perfect balance of power between the vendor and the buyer, enforced by contractual agreement, there are also models developed so as to minimise the vendor's total annual cost subject to the maximum cost that the buyer may be prepared to incur (e.g. Lu, 1995). However, it is clear how a partnership may be strategically useful to both sides, and how it can improve the inventory management policies (i.e. reduce the costs) by loosening some constraints. This is described in Hill (1997), who introduced a reference model in an integrated perspective. He showed how the *system* as a whole can enjoy significant improvements without any relevant loss for the parties.

To summarise, the literature highlights the theoretical optimality of $(s, S)$ policies, but it is inconclusive on a lot of issues regarding the levels of inventory, variables to which the results of the model are very sensitive. Moreover, the benefits of a collaboration between companies and suppliers are not taken into account: in other words, suppliers' costs are not considered a key variable. On the other hand, the models that allow for profitable interaction between the parties often lack the flexibility and the practicality of general $(s, S)$ policies, and provide complex formulas of order quantities.

As we shall see, CS seeks to use the best of both approaches.

## 3. Considerations on CS fundamentals

The basic idea of CS consists in the fact that the physical inventory of raw material resides within the company, ranging freely in quantity between a minimum required level $s$ and a maximum permitted level $S$. In addition, under the agreement, the material is formally purchased only at the moment of its consumption. In other words, even if the raw material is stored in the company's warehouse, it is still "owned" by the supplier. Let us see what changes in the cost structure this policy entails.

We may think of the per unit inventory cost $h$ as driven by two main components: a financial one ($h_{\text{fin}}$) and a storage one ($h_{\text{stock}}$). The financial part regards the opportunity costs a firm incurs while investing financial resources in producing a good. The operational component has to do with the pure storage and movement costs, insurance costs, etc. Under a typical supply agreement, these costs are borne as indicated in Table 1. That is, each firm, the supplier and the company in turn, sustains the entire inventory cost (both the financial and the storage components) as determined by the firms' endogenous characteristics while the material is stored in their warehouse. In the simplest terms, no inventory cost is sustained by the supplier after delivering the goods, and no cost is borne by the company before the delivery.

Finally, it should be noted that $h_{\text{c,fin}} + h_{\text{c,stock}}$ is generally greater than $h_{\text{s,fin}} + h_{\text{s,stock}}$, mainly because of the financial component, which increases as it goes down the supply chain. The different situation brought about by CS is outlined in Table 2.

As can clearly be seen, the main difference is to be found in the case where the material has already been delivered to the company. In fact, the company incurs the storage cost, given that the material is located in its warehouse, but it does not yet sustain the financial cost. In fact, given that a good is formally purchased only after its

Table 1
Relevant inventory costs in traditional agreements

| | | Position of raw material | |
|---|---|---|---|
| | | Supplier | Company |
| Relevant costs | Supplier | $h_{\text{s,fin}} + h_{\text{s,stock}}$ | 0 |
| | Company | 0 | $h_{\text{c,fin}} + h_{\text{c,stock}}$ |

Table 2
Relevant inventory costs under CS policy

| | | Position of raw material | |
|---|---|---|---|
| | | Supplier | Company |
| Relevant costs | Supplier | $h_{s,fin} + h_{s,stock}$ | $h_{s,fin}$ |
| | Company | 0 | $h_{c,stock}$ |

consumption, the supplier is still bearing the financial opportunity cost.[1] Thus, while calculating the total cost for the system, we may reasonably assume that the storage component $h_{stock}$ of the total inventory cost may be considered as more or less identical for the supplier and the company, i.e. $h_{c,stock} = h_{s,stock}$. As a consequence, referring to the same average stock level, the total storage cost of the system is lower in the CS case (as we assume $h_{c,fin} > h_{s,fin}$), even if a part of the cost is "shifted" onto the supplier. However, the supplier perceives some advantages as counterpart: (i) the average quantity of the material stored in his own inventories decreases and, consequently, (ii) space is available to allocate other items; finally, (iii) the supplier may manage his production plan more flexibly as it is not constrained by closed-orders. On the other hand, the company "sees" a lower per unit inventory cost, that is, only $h_{c,stock}$ instead of the entire $(h_{c,fin} + h_{c,stock})$.[2]

Furthermore, it should be noted that there is no longer any administrative cost for placing an order, as, in fact, there is no longer any order.[3] It should be added that, in some applications of CS strategy, the item carrier may also be involved in the partnership, as it may delivery goods from different suppliers and update the information on inventory levels.

To give substance to our arguments, we will try to show more formally how adopting the CS policy improves the performance of the company/supplier system; more specifically we will compare it to Hill's model (Hill, 1997) while dealing with Goyal's (1988) classic example. The following numerical values and parameters are taken into account:[4]

$A_1$ = supplier's set-up cost = 400\$,
$A_2$ = cost of placing one order = 25\$
$h_1$ = supplier's inventory cost/unit = 4\$,
$h_2$ = company's inventory cost/unit = 5\$,
$D$ = demand rate = 1000 units/year,
$P$ = production rate = 3200 units/year
$n$ = number of deliveries per lot.

The 'delivery lead-time' is set equal to zero. According to Hill (1997), the order quantity $q^*$ that minimises $C(n, q)$, the average joint cost per unit of time, is given by

$$q^* = \sqrt{\left((A_1 + nA_2)\frac{D}{n}\right)\left(h_1\left(\frac{D}{P} + \frac{(P-D)n}{2P}\right) + \frac{h_2 - h_1}{2}\right)^{-1}}.$$

Applying the classic EOQ solution (Buffa and Sarin, 1987), we would get an optimal EOQ of 100 items. Hill's solution identifies a minimum cost of 1903\$ in correspondence to a modified EOQ = 110 and an EPQ = 550, thus providing the condition of minimum cost for the whole buyer–vendor system.

In order to compare Hill's model to CS, a series of tests has been run.

If the cost function is modified according to the CS policy as described in Table 2, (i.e. removing company order costs and splitting the inventory costs into a financial component and in a storage one of equal amount),[5] Fig. 1 is obtained: it represents the total average joint costs—on the basis of a 20-year period[6]—depending on the $S$ level ($s$ is set to zero, because of the null lead-time assumption).

[1] Obviously, we are not considering the delay between the purchase and the payment by the company. Still, our argument holds *ceteris paribus*.

[2] Admittedly, the average quantity stored in its own inventories may be larger.

[3] Because of the EDI interface between the company and the supplier.

[4] In the model there is no transportation cost. This may be realistic, for instance, when geographical proximity is assumed. Goyal assumes linear costs to be usual.

[5] Results depend on this assumption, but a basic sensitivity revealed how simulation main outcomes hold for a wide enough range of $h_{stock}$ and $h_{fin}$.

[6] This assumption is made in order to obtain better quality graphs, but it does not influence the indications offered by the analysis carried out.

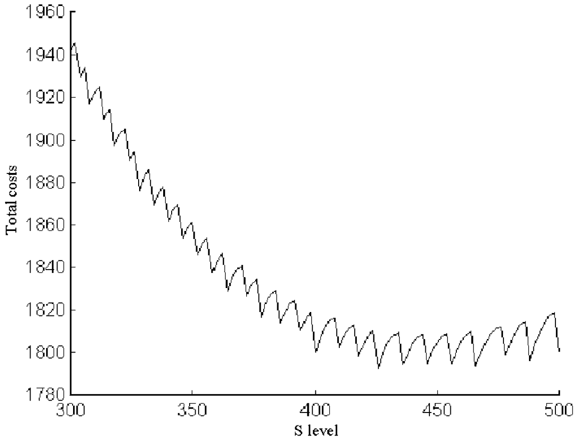


Fig. 1. Total joint costs in the CS policy depending on $S$.

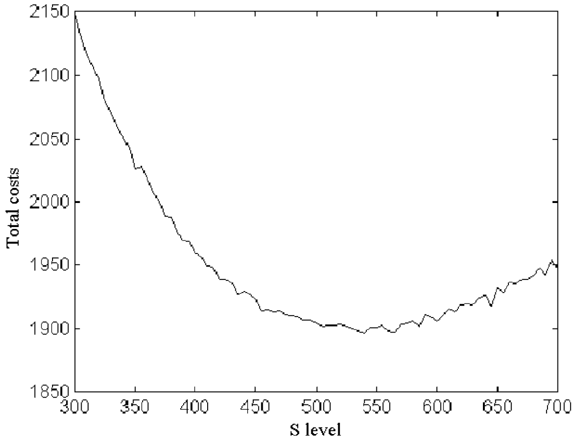


Fig. 2. Hill's model costs depending on $S$ in the case of a non-deterministic demand.

At the present stage, it may even be misleading to take into account all the variables at stake which may determine the supplier deliveries in between $s$ and $S$. Thus, we assumed that when $s$ is reached, an $(S-s)$ quantity is immediately delivered by the supplier. The minimum cost obtained is 1793\$, in correspondence to $S = 426$. Adopting CS policy and the cost savings implicit in it, it is therefore possible to achieve a 6% reduction in the total cost. Moreover, neither the supplier's allowed flexibility and the company's reliability in guaranteeing service levels were quantified in monetary terms, in spite of their indisputable value.

Even if it is quite obvious to obtain a reduced cost with respect to Hill's model (as some costs are lower), the test results show that this CS performance depends on the $S$ level fixed. Thus, a first insight is possible: when the company and its supplier bargain the CS agreement, they must take into account that the convenient $S$ level is to be identified for the entire system.

Now, let us remove the hypothesis of deterministic demand and divide the year into 100 periods, each with a demand profile sampled from the normal distribution $d \sim N(10, 1)$. In such a way, and still considering Hill's model, the possibility of incurring a stockout is introduced and a more realistic market situation is taken into account. Simulating this policy over 50 years, we obtain the cost curve of Fig. 2. The average cost is slightly lower (1897\$) than Hill's model performance for deterministic demand, but what matters is that in this point an average demand of nine items per year remains unsatisfied. The costs determined by the shortage event were not considered in the calculation but, especially when the company considers the service level as a strategic key for gaining competitive advantages, they may be of great relevance. As far as stockout events are concerned, it is also interesting to analyse the average number of items undelivered by the company per year, while varying the $S$ value (Fig. 3). Of course, the magnitude of the phenomenon may be decreased while increasing the $S$ level (thus incurring additional storage costs), but the greater the level, the lower the possibility of further service improvement.

In conclusion, it may argued that, with the same total cost, the CS policy allows the company to keep an $s$ level of even more than 20 items, thus reducing stockout risk. The results proposed suggest a second insight. For a fluctuating demand, the CS policy allows a higher service level (i.e. less shortage events) at acceptable costs. Once again, this performance depends on the $(s, S)$ levels adopted. The same positive results described above may be extended to the common case of a variable lead time in the supplier deliveries. In fact, generally speaking, $s$ level may help the company in facing both demand variations and lead time

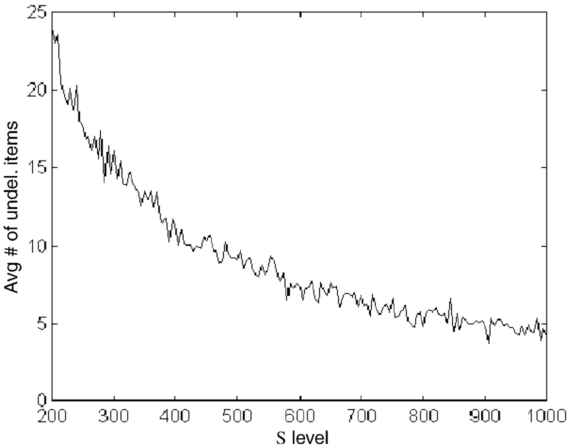


Fig. 3. Common supplier–vendor model and non-deterministic demand: undelivered items because of stockout as a function of $S$.

uncertainties. Furthermore, CS policy, thanks to EDI interface, allows the supplier to perceive directly the trend of the demand, thus reducing or at least stabilising the lead time.

These considerations reinforce our initial belief that the importance that CS is acquiring in practical terms, although it has often been ignored in literature, rests on a rational basis. However, several practical issues remain unresolved. Addressing them is the subject of the following section, starting from the case observed.

## 4. The industrial case

Having shown that CS could be the best choice does not solve the implementation problems. Is CS the best solution for every item? What are the forces that determine the levels of $S$ and $s$? Once $S$ and $s$ have been fixed, how does the supplier manage his degrees of freedom acquired through CS? To provide an answer to these questions, we will consider general insights and the case study together.

The case analysed refers to an Italian company, which manufactures components for the automotive industry (braking systems). The company saw a fast growth in the recent past, roughly doubling both gross income and personnel from 1995 to 1999. The most recent evolution gave it an industrial presence in Japan and a consolidated position as a supplier to the most important European manufacturers of cars, industrial vehicles and motorbikes. The industrial activity also expanded into the racing sector, strategically identified in dedicated areas of the productive units. The production covers both the Original Equipment and the After Market demand. The relevant performances obtained are also to be related to the effective introduction of techniques such as TQM, Just in Time and Kaizen, flexibility in assembling and production, personnel motivation. As far as supplies are concerned, the company recently introduced the CS technique and some interesting questions arise from its experience.

The first insights we get refer to the choice of the items and suppliers to be involved in the CS program. The suppliers involved in the project must be among the most active and critical, starting from considerations on the type and/or quantity of pieces supplied together with the economic relevance of their supplies. Thus, an initial step consists in identifying:

- The most attractive items for CS management (e.g. criticality for the assembling tasks, strategic value of the system to which the component pertains etc.): such an analysis also leads to the identification of the suppliers to be contacted.
- According to the resulting list, the most qualified suppliers are selected (e.g. on the basis of their turnover with respect to the company, on their performance in quality assurance and stability in delivery time) for the first implementation of the CS supply.

Afterwards, a technical comparison between the company and the supplier is necessary to identify the salient parameters for the items to be provided under CS management (e.g. safety stocks, lead times, packaging conditions, transport quantity). Consequently, the minimum and maximum item levels (i.e. $s$ and $S$) to be guaranteed over time are jointly fixed. The forces that constitute the variables at stake, while suppliers and company negotiate the level of $s$ and $S$, are summed up in Table 3.

Table 3
Desired (s, S) levels and related motivation for the supplier and the company

| | s level | S level |
|---|---|---|
| *Supplier* | ↓Keep as low as possible<br>It represents a capital "frozen" in the company's warehouse, which determines an opportunity cost | ↑Keep as high as possible<br>It consists in a space offered by the company to stock products, thus freeing space in own warehouses. Consequently, production flexibility may be increased. |
| *Company* | ↑Keep as high as possible<br>It is a sort of safety stock which enables a higher service level, whereas its economic burden is borne by the supplier | ↓Keep as close as possible to s level<br>There is a need for limiting the space occupied by the products and for reducing the problems linked to their management and handling |

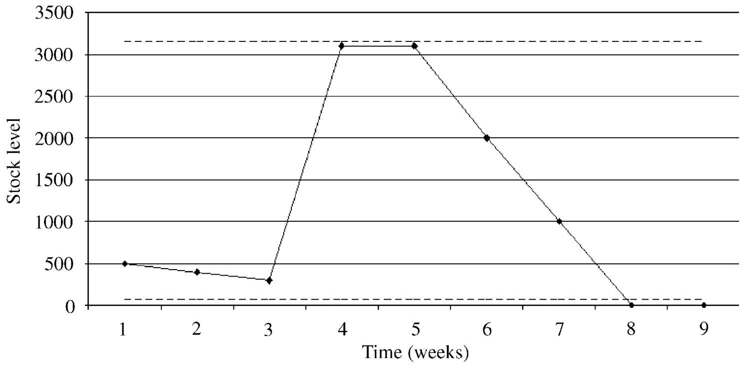


Fig. 4. Inventory level of an item through time (dashed lines indicate $s$ and $S$ level)

On the basis of the industrial case, one field to be explored is related to the supplier's behaviour while maintaining the stock within the ($s$, $S$) range. In fact, two limiting conditions may be hypothesised.

In the first case, the supplier keeps his stock close to the upper level $S$ or, alternatively, he may fill the company warehouse up to the $S$ level and consequently wait for the stock erosion, due to demand requirements, down to level $s$ (Fig. 4). In such a situation, the profile of the company inventory is similar to the classic behaviour of a continuous-review model, with fixed order point and order quantity. E.g., this situation may occur when $(S - s) \approx \text{EPQ}$. According to Table 2 motivations, the supplier may produce according to his EPQ, but his own warehouses are freed, as space is provided by the company. It is also possible to understand that, in these cases, the company has not fully used its bargaining power (or the supplier has some…), since $S$ is comparatively high even if the item does not seem to have any strategic relevance despite its high rate of consumption.

In the second limiting condition, the supplier may decide to keep the company's inventory close to the $s$ level, thus incurring the risk of penalty payment due to the lower level violation, but reducing his economic exposure (e.g., if $\text{EPQ} \ll S$).

Of course, an intermediate behaviour is represented by the suppliers delivering quantities so that the inventory level fluctuates between the two $s-S$ limits. This situation, graphically described in Fig. 5, refers to the interesting case of a component subjected to frequent delivery and irregular, though intensive, consumption: the supplier continuously switches within the $s$ and $S$ levels, thanks to a flexibility in production volume and a "flat" behaviour of its characteristic EPQ curve. Also in

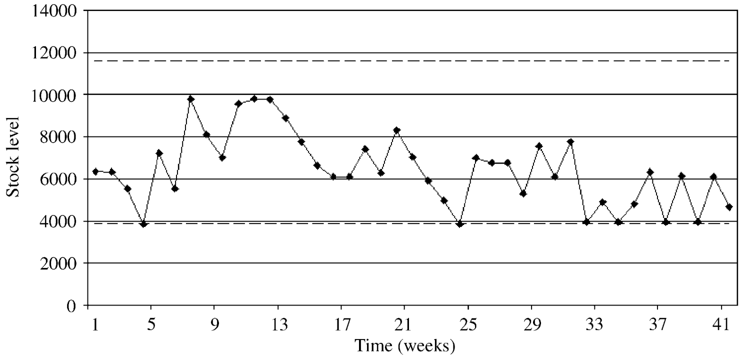


Fig. 5. Inventory level of an item through time (dashed lines indicate $s$ and $S$ level).

this case, the risk of shortage could arise in some situations. The curve shows the stock level of a real item of the industrial case observed. It is a very strategic one (this justifies a reasonably high $s$ level), and is produced by a very close supplier, extremely flexible, and with a number of important customers. In fact, the erratic profile of the supply curve not only points to several factors relative to the specific item, but is also an index of the increased supplier flexibility in deciding his production plan while trying to cope with customers' needs. In the absence of precise and imperative production order and time deadline from a given company, the supplier has a greater degree of freedom to organise and adapt his manufacturing resources. This item also highlights another important variable in CS policy: item dimension. The high number of $s$ and $S$ is also an index of the small dimensions of the item involved. It may seem trivial, but this is another important factor that has to be taken into account while deciding inventory levels.

The problem of transported quantities is also relevant, this being related in turn to the geographical position of the supplier with respect to the company location. Generally speaking, distant suppliers aim to fill the stocks up to $S$ level (large quantities transported, also by ship and/or train), whereas the closest suppliers (which deliver small quantities by local truck transportation) may keep a stock close to $s$ level and/or show the irregular pattern of deliveries already described.

According to the industrial experience analysed, some other issues emerge:

- Items to be included in the CS programme are those characterised by a constant consumption (“open order” provisioning).
- Items of potential interest for CS pertain to standard production, but they are subjected to a “close order” provisioning system. Coming under the CS management, they will pass to an “open order” provisioning.
- Items to be excluded from CS management are non-standard products and prototypes (e.g. in the industrial case under examination, items for racing models are excluded, together with production for prototypes and first-sample supply).
- The minimum level $s$ may be roughly estimated as the safety stock which enables the company to cover the production of a period the length of which depends on the lead time of the supplier.

The agreement between the company and the supplier may include further obligations, such as:

- The agreed lead time in case of sudden demand peaks for the company.
- The level of the safety stock the supplier should maintain in his own depots, taking into account the provisioning time of the item considered. This parameter may also influence $s$ and $S$ values.

- The type and capacity of the pallets for delivery, as $s$ and $S$ values are an integer multiple of it. This parameter is also to be fixed to interface CS standards with the kanban system.
- The company may agree to pay for the goods stored in its warehouse, even if it has not consumed them yet, after a given amount of time.

Some of the above topics are prudentially introduced when starting the CS management of items: they can be removed once the system parameters have been conveniently arranged. Nevertheless, safety stocks and planned lead times may be also providential measures to meet unpredictable or turbulent market demand, especially in the case of suppliers that lack production flexibility. It must be highlighted how the final agreement generally involves further parameters which are linked more closely to fiscal and legal regulation. Thus, it may be difficult to discuss them exhaustively, as they may differ in an international context. Though, it is interesting to comment on some of them:

- The eventual dispatching of items exceeding $S$ level allows the company to send them back to the supplier without incurring transport costs.
- Shortage costs may be included in the agreement as a penalty for the supplier, if the event occurs because of failure to respect the $s$ level. Such a penalty may be large enough to cover the entire economic damage caused to the company.
- Every single item pertaining to a stock is the property of the supplier until the company draws on it for production.

## 5. Conclusions

This paper gives the account of an innovative inventory policy called consignment stock. Its rapid diffusion and the absence of any important reference to it in the scientific literature led us to ask ourselves whether CS was just a fad or whether it had some value.

We showed how, by combining the practicality of $(s, S)$ policies with the "system approach" of joint-profit maximising models, CS is able to outperform the usual inventory models. Not only does CS allow savings in inventory costs, but it also entails several complementary intangible advantages, such as a higher degree of flexibility, an increased service level in turbulent environments, a reinforced and reliable relationship between companies and suppliers. Even Authors that argue that CS may be harmful for the supplier from a purely economic point of view, recognise the strategic importance of the relational rents stemming from CS (Ferrozzi and Shapiro, 2000). Hence, the diffusion of CS has a clear rationale behind it, which at present we have shown by simulation experiments.

Moreover, thanks to a case study, we have provided some insights into tactical issues that firms have to address once they decide to adopt the CS policy. We believe we have provided reasons that show *why* CS is catching on as well as *how* it is and should be implemented.

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