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Modelling an industrial strategy for inventory management in supply chains: the 'Consignment Stock' case

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Stock control in Supply Chain management is of concern here, particularly an industrial practice observed in the automotive manufacturing context and defined as 'Consignment Stock' (CS). To understand the potentiality of CS policy, an analytical modelling is offered that refers to the problem of a single-vendor and single-buyer productive situation. A comparison with the optimal solution available in the literature is also shown. The conclusion proposes a method that is useful in identifying those productive situations where CS might be implemented successfully. Results show how CS policy might be a strategic and profitable approach to stock management in uncertain environments, i.e. where delivery lead times or market demand vary over time.

1. Introduction

Several models can be found in the literature for inventory management and control. More recently, increased interest in Supply Chain topics has seen researchers address the problem of cooperation between the buyer and vendor, i.e. the two parties directly interacting in the complex supply mechanism (e.g. see the conclusions in Goyal and Gupta 1989). For isolated situations and deterministic demand, it is shown how the optimal solution can be identified by the Economic Order Quantity (EOQ) model. When applied to productive environments, it allows the vendor to calculate the Economic Production Quantity (EPQ), although it might be significantly different from the buyer's EOQ. As a result, the two parties enter into negotiation to reach a compromise that involves the price per item and the size of the batch to be supplied. Of course, the negotiation result depends on the relative strength of the two parties, creating the basis for an agreement which is optimal for neither the buyer nor the vendor (Banerjee 1986). From the vendor's point of view, a discount policy may be adopted to encourage the buyer to purchase the material quantity, which maximizes the profit, i.e. a quantity close to the EPQ (e.g. Lal and Staelin 1984, Monahan 1984, Lee and Rosenblatt 1985, 1986, Banerjee 1986).

According to the Joint Economic Lot Size (JELS) model (Goyal 1977), the most competitive approach consists in minimizing the sum of the costs of both the buyer and vendor. The JELS model may be generalized, introducing the hypothesis of

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the vendor's discrete production (Banerjee 1986) and removing the hypothesis of the vendor's need for selling batch by batch (Goyal 1988).

An essential factor in these models is that the vendor knows the demand and the basic costs of the buyer (i.e. material holding and order emission costs). According to Monahan (1984), the buyer's costs can be estimated by a simple analysis of the size of the orders previously emitted.

More recently Hill's (1997, 1999) contributions focused on a model that can minimize the total costs per year of the buyer–vendor system. The basic assumption is that the vendor only knows the buyer's demand and order frequency. Consequently, the model can be applied where there is cooperation between the two parties, regardless of the possibility that they may belong to the same corporation or company.

2. Hill's model

Generally, the vendor's production is organized in batches, thus incurring set-up costs. Each batch is delivered to the buyer by a certain number of transport operations, also made while production is running. Each transport operation determines a fixed cost, i.e. an order emission cost. The problem of the optimal number of deliveries is of significant relevance and it has been widely discussed with reference to Hill's model (Hill 1999, Hoque and Goyal 2000). Both parties incur material holding costs depending on different rates and the time for which materials are stocked. The buyer uses the products purchased according to market demand. Thus, the following notation can be introduced:

A_1 batch set-up cost (vendor), e.g. 400 (\$/set-up),
A_2 order emission cost (buyer), e.g. 25 (\$/order),
h_1 vendor holding cost per item and per time period, e.g. 4 (\$/item·year),
h_2 buyer holding cost per item and per time period, e.g. 5 (\$/item·year),
P vendor production rate (continuous), e.g. 3200 (units/year),
D demand rate seen by the buyer (continuous), e.g. 1000 (units/year),
n number of transport operations per production batch,
q quantity transported per delivery, from which the production batch size $Q = n \cdot q$,
C average total costs of the system per time unit, being a function of n and q.

The values reported refer to Goyal's example (1988), adopted as reference. It is also assumed that $P > D$ and $h_2 > h_1$. The former hypothesis is obvious, while the latter is linked to the common opinion that an item increases its value while descending the distribution chain. As a consequence, goods are generally kept in the vendor's warehouses until the buyer's request for a further shipment. Figure 1 shows the trend of the stock levels in the case of five shipments per batch produced (two of the five shipments planned take place while the vendor produces a single batch). In this case: $Q = 550$ (items), $n = 5$ and $q = 110$ (items).

According to Hill's model, the total costs are:

$$C = (A_1 + nA_2)\frac{D}{nq} + h_1\left(\frac{Dq}{P} + \frac{(P-D)nq}{2P}\right) + (h_2 - h_1)\frac{q}{2}. \quad (1)$$

Function C may be differentiated with respect to q, thus obtaining $C'(q)$ function. Once $C'(q)$ is set equal to zero, the batch size q^* able to minimize the

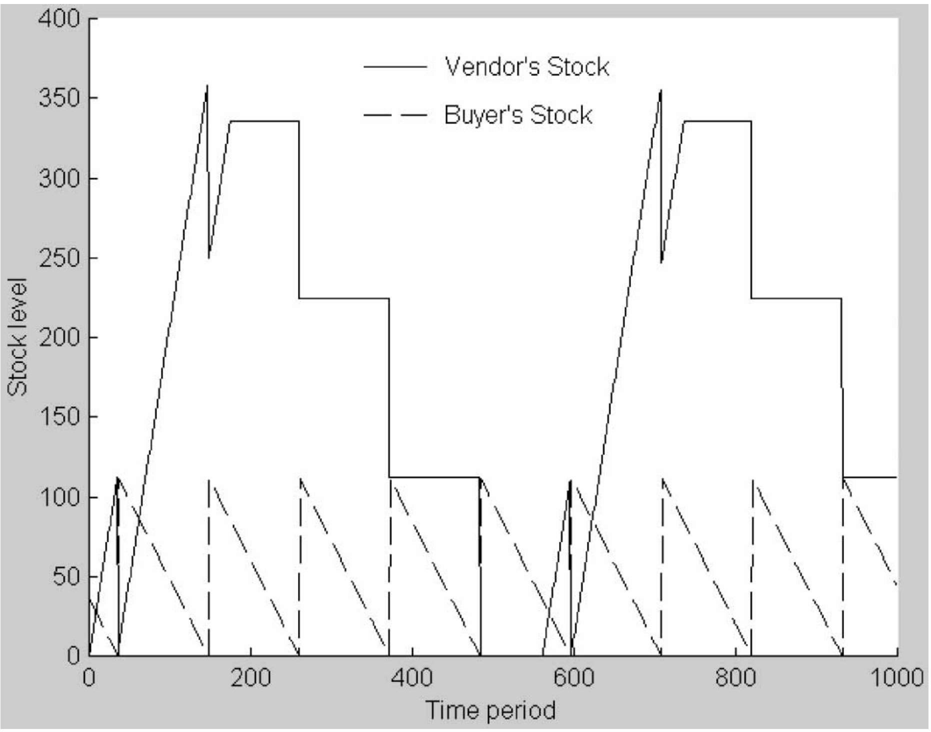

Figure 1. Hill's model: level of stocks at the buyer and vendor inventories.

total costs C is found:

$$q^* = \sqrt{\left((A_1 + nA_2)\frac{D}{n}\right) \Big/ \left(h_1\left(\frac{D}{P} + \frac{(P-D)n}{2P}\right) + \frac{h_2 - h_1}{2}\right)} \tag{2}$$

for a minimum cost $C(q^*)$ equal to:

$$C(q^*) = 2\sqrt{\left((A_1 + nA_2)\frac{D}{n}\right)\left(h_1\left(\frac{D}{P} + \frac{(P-D)n}{2P}\right) + \frac{h_2 - h_1}{2}\right)}. \tag{3}$$

3. Consignment Stock strategy

The main strategic finding implicit in Hill's model is that the cooperation between the buyer and vendor gives a far greater benefit than a non-collaborative relationship. A different policy, observed and applied in a manufacturing company, will be described below. According to industrial practice, it will be defined as Consignment Stock (CS) and it requires a continuous exchange of information between the two parties. The most radical application of CS may lead to the suppression of the vendor's inventory, as this party will use the buyer's warehouse to stock material. This warehouse is close to the buyer's production line so that the material may be picked up when needed. Furthermore, the vendor will guarantee that the quantity stored in the buyer's warehouse will be kept between a maximum level (S) and a minimum one (s), also supporting any additional costs induced by stock-out conditions. The buyer will take from the store the quantity of material necessary to cover the production planned and the vendor will be paid up to a daily frequency, thus transmitting to the vendor fresh and immediate information on the consumption trend. The following brief comments describe some of the various tasks implicit in the CS policy.

The buyer:

- has a constantly guaranteed minimum stock level, i.e. s;
- does not have to take care of order emission (minus administrative costs);
- pays for goods only when they are effectively used (minus the quantity of 'frozen' capital); and
- does not pay for capital-linked holding costs, as they are chargeable to the vendor.

The vendor:

- has access to the final demand profile, thus by-passing the filter determined by the buyer's orders, as occurs in the classic approach;
- has the opportunity to empty his warehouse, thus using it for other tasks (storing raw materials, installing additional productive capacity etc.). Of course, the extent of this advantage depends on the relative values of level S, the production rate P and the order size Q; and
- may organize his production campaigns differently, being less closely linked to the buyer's requirements.

In addition, another important benefit for the entire supply chain system must be highlighted. It is well known (e.g. Chen *et al.* 2002) that the strategic partnership between the buyer and vendor (as implicit in the CS approach) allows the reduction or elimination of the bullwhip effect, i.e. of the increase of demand variability as one moves up a supply chain.

Nevertheless, the most evident difference between Hill's model and the CS approach lies in the location of the stocks, which are preferably located in the vendor's warehouses in the first case, instead of the buyer's, as CS management implies. It is evident that a deterministic environment implies the optimal performance of Hill's model, i.e. a stable demand together with predictable lead times works in favour of a policy invoking the maintenance of goods where holding costs are lower and transport may be delayed until goods are required. The following sections will investigate the influence of demand and lead time variability on the performance of the two policies.

A brief comment on s and S levels is needed, as the buyer's and vendor's interests are conflicting ones.

The vendor:

- will try to set the s level as low as possible, so as to reduce the cost of the safety stock that he himself must guarantee; and
- will try to set the S level as high as possible, so as to exploit his production capacity until the buyer's warehouses are full.

The buyer:

- will try to set a higher s level, so as to reduce the stock-out probability (even if penalties are chargeable to the vendor); and

- will try to set the S level as close to the s level as possible, so as to reduce the space occupied and the relative costs linked to investment in structures.

Of course, the need for negotiating the s and S levels represents an opportunity for profitable cooperation between the two parties.

4. Analytical model of CS policy

As in Hill's model, the vendor incurs set-up costs and produces according to batches. Deliveries require various transport operations, some of which are carried out while production is running (figures 1 and 2). The buyer and/or the supplier are subject to a fixed cost for order emission and transportation, this being assumed as independent of the quantity q to be transferred. Both of the parties incur holding costs, although at different rates.

When applying the CS technique in its simplest form, items are delivered to the buyer whenever the product level in the vendor's stock reaches quantity q, thus obtaining the profiles shown in figure 2 ($Q = 512$ (items), $n = 4$ and $q = 128$ (items)).

The CS model described in figure 2 also matches the industrial case, which originated the present study. The vendor's behaviour proposed in figure 2 was generally observed as well as being a 'natural' one. In fact, a strategic advantage of the vendor lies in the use of the buyer's warehouse space. Thus, the supplier aims to keep his stock level as low as possible, according to the limitations imposed by the S level. Of course, various ways of behaviour on the part of vendors was observed, but the

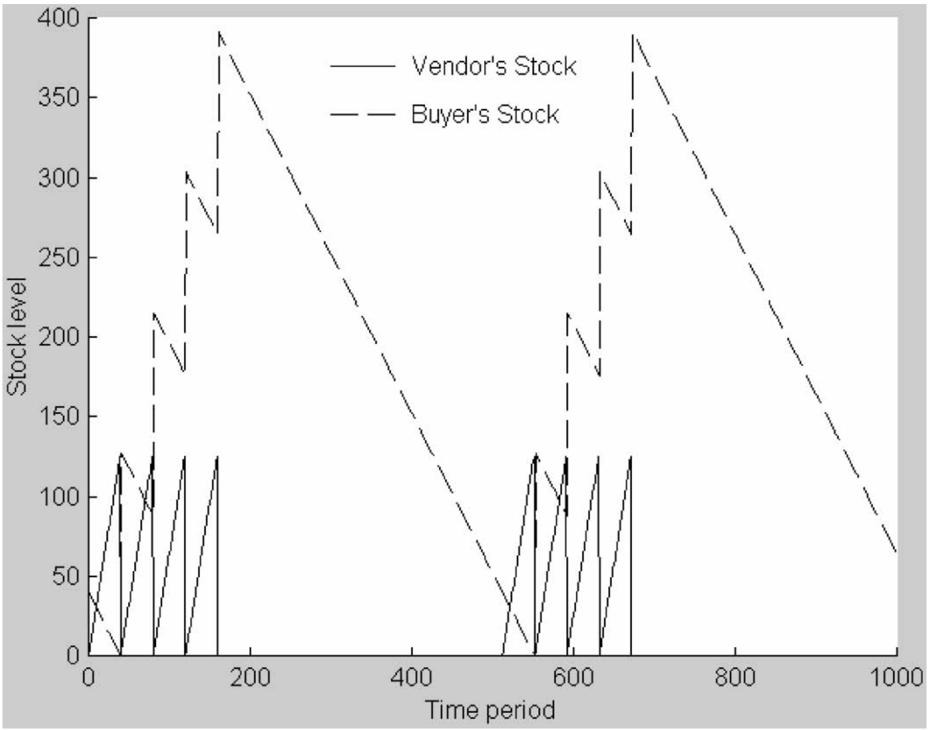

Figure 2. CS model: level of stocks at the buyer and vendor inventories.

one adopted is quite significant, as it also emphasizes the possible impact of the CS approach on the buyer's stocks. Another feature worth noting about the industrial case observed, is that the s level is frequently set to zero.

The vendor's average costs per year have two contributing factors:

$$\text{Set-up cost}: C_s^v = A_1 \frac{D}{n \cdot q} \tag{4}$$

$$\text{Holding cost}: C_m^v = h_1 \cdot \left(\frac{q \cdot D}{2 \cdot P} \right). \tag{5}$$

In formula (5), contribution $qD/2P$ is the product between the average quantity in the store, $q/2$, and the time D/P during which the level of the vendor's stock is other than zero. Buyer's costs are:

$$\text{Order emission cost: } C_e^b = A_2 \frac{D}{q} \tag{6}$$

$$\text{Holding cost: } C_m^b = \frac{h_2}{2} \left(n \cdot q - (n-1) \cdot \frac{q}{P} D \right). \tag{7}$$

The total holding cost is determined by h_2 multiplied by the average inventory level, as obtained by basic geometric considerations, being equal to the average between the maximum and minimum level (zero). The total costs for the system are:

$$C = (A_1 + nA_2) \cdot \frac{D}{n \cdot q} + h_2 \left(\frac{D \cdot q}{P} + n \cdot q \frac{P - D}{2 \cdot P} \right) - (h_2 - h_1) \left(\frac{q \cdot D}{2 \cdot P} \right) \tag{8}$$

and they can be differentiated with respect to q and setting the derivative to zero, thus obtaining the optimal quantity q^* which minimizes the total costs themselves:

$$q^* = \sqrt{\frac{(A_1 + n \cdot A_2)(D/n)}{h_2((D/P) + n(P-D)/(2 \cdot P)) - (h_2 - h_1)(D/2 \cdot P)}} \tag{9}$$

giving a minimum cost equal to:

$$C(q^*) = 2\sqrt{\left((A_1 + nA_2) \frac{D}{n} \right) \left(h_2 \left(\frac{D}{P} + n \frac{P-D}{2P} \right) - (h_2 - h_1) \frac{D}{2P} \right)}. \tag{10}$$

Of course, the maximum level of the vendor's stock is equal to q, while the buyer's may be evaluated by the following:

$$Mag_{\max}^b = n \cdot q - (n-1) \frac{q \cdot D}{P}. \tag{11}$$

According to the behaviour adopted by the vendor (figure 2), the $Mag_{\max}^b$ and S values clash, or $S > Mag_{\max}^b$.

4.1. *Numerical example*

When adopting data from Goyal's example, the formulae discussed above lead to the results shown in figure 3. The minimum of the total costs is found for 2034.9 (\$/year), for $n = 2, 4$ and 6 versus the maximum level S of the buyer's inventory.

This makes it possible to calculate the minimum total cost with reference to the number of shipments, too. Thus, the problem of the optimal number of deliveries to be carried out is numerically solved, leaving its analytical solution to further research.

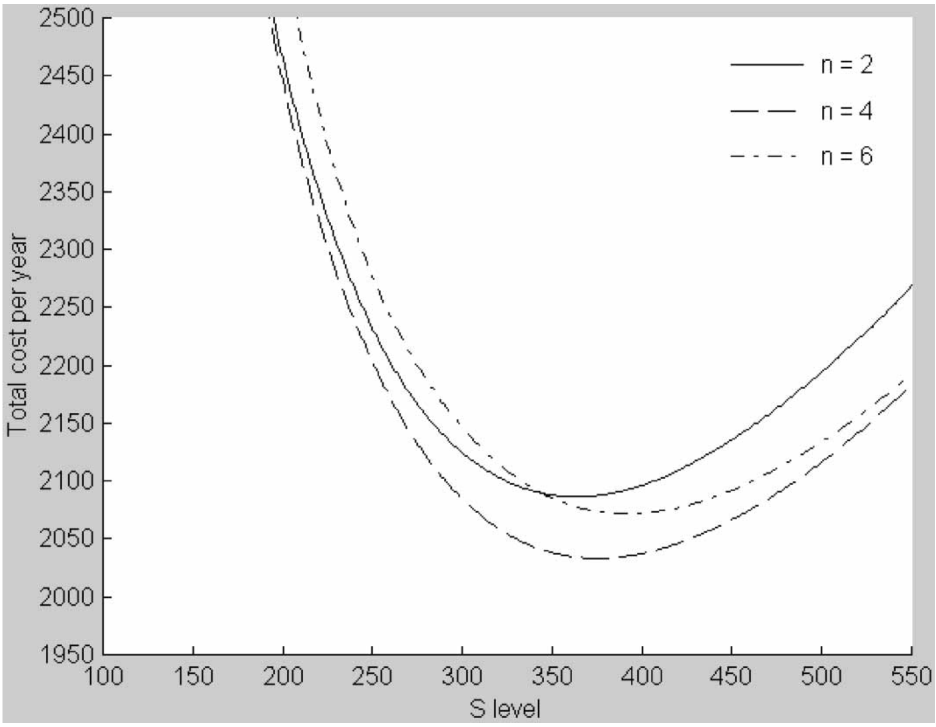

Figure 3. Total costs for CS policy with different n and S values.

5. CS model for delayed deliveries

The analysis of the basic CS model highlights a possible inefficiency of the model itself, due to the relevant value that the maximum level of the buyer's inventory may reach, even if for limited periods. A possible solution is offered by delaying the last delivery until the moment when it no longer determines a further increase in the maximum level already reached. The situation is described by figure 4, where R is the lapse of time introduced to delay the last delivery.

The vendor's average costs are the sum of two factors:

$$\text{Set-up cost: } C_s^v = A_1 \frac{D}{nq} \tag{12}$$

$$\text{Holding cost: } C_m^v = h_1 \left(\frac{qD}{2P} + q \frac{P-D}{nP} \right), \tag{13}$$

where $(q \cdot D)/(2 \cdot P)$ is the contribution of the n triangles, and $q.(P-D)/(n \cdot P)$ comes from the area corresponding to the delayed q. The buyer's costs become:

$$\text{Order emission cost: } C_e^b = A_2 \frac{D}{q} \tag{14}$$

$$\text{Holding cost: } C_m^b = h_2 \left(\frac{Dq}{P} + nq \frac{P-D}{2P} - \frac{qD}{2P} - q \frac{P-D}{nP} \right). \tag{15}$$

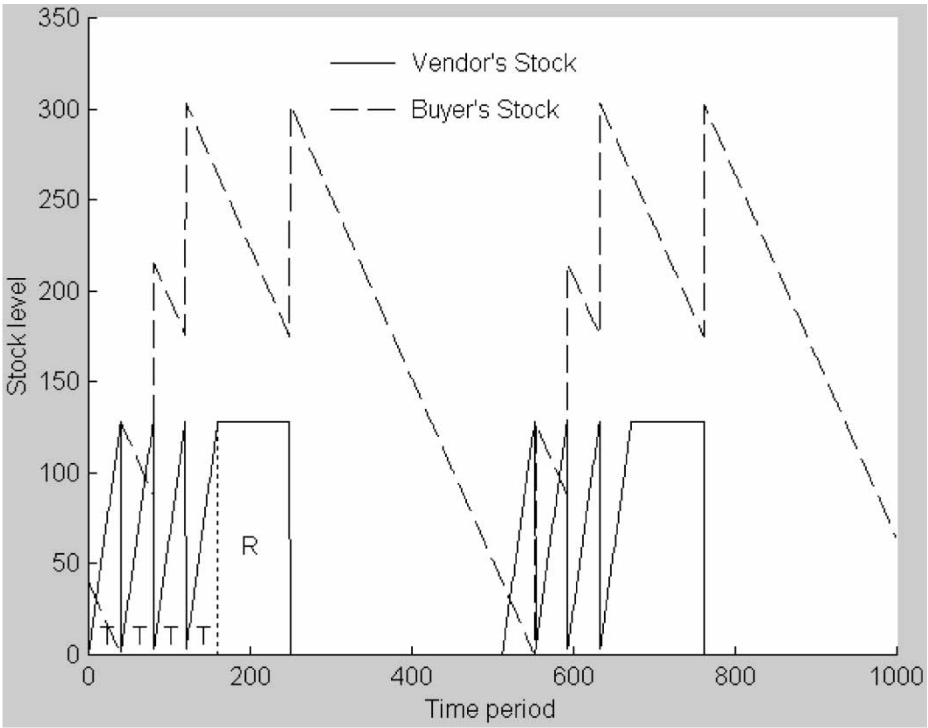

Figure 4. Buyer's and vendor's inventory levels when delaying the last delivery.

Once again, total system costs may be evaluated:

$$C = (A_1 + nA_2)\frac{D}{nq} + h_2\left(\frac{Dq}{P} + nq\frac{P-D}{2P}\right) - (h_2 - h_1)\left(\frac{qD}{2P} + q\frac{P-D}{nP}\right) \tag{16}$$

and setting the derivative to zero, the minimizing quantity q^* is found:

$$q^* = \sqrt{\frac{(A_1 + nA_2)(D/n)}{h_2((D/P) + n(P-D)/(2P))) - (h_2 - h_1)((D/2P) + (P-D)/(nP))}} \tag{17}$$

offering a minimum total cost $C(q^*)$ equal to:

$$C(q^*) = 2\sqrt{\left((A_1 + nA_2)\frac{D}{n}\right)\left(h_2\left(\frac{D}{P} + n\frac{P-D}{2P}\right) - (h_2 - h_1)\left(\frac{D}{2P} + \frac{P-D}{nP}\right)\right)}. \tag{18}$$

The maximum level of the buyer's stock is:

$$Mag^b_{\max} = (n-1)q - (n-2)q\frac{D}{P}. \tag{19}$$

The model discussed may be regarded as a particular example of a more general case, i.e. the model with k delayed deliveries (CS-k). In this case, the analytical relationships become:

$$\text{Set-up cost: } C_s^v = A_1 \frac{D}{nq} \tag{20}$$

$$\text{Vendor's holding cost: } C_m^v = h_1 \left(\frac{qD}{2P} + q \frac{P-D}{nP} \frac{(k+1)k}{2} \right), \tag{21}$$

where the term $((k+1)k)/2$ equals $\sum_{j=1}^{k} j$.

$$\text{Order emission cost: } C_e^b = A_2 \frac{D}{q} \tag{22}$$

$$\text{Buyer's holding cost: } C_m^b = h_2 \left(\frac{Dq}{P} + nq \frac{P-D}{2P} - \frac{qD}{2P} - q \frac{P-D}{nP} \frac{(k+1)k}{2} \right). \tag{23}$$

The total costs of the system are given by the sum of the (20–23) contributions, thus obtaining:

$$C = (A_1 + nA_2) \frac{D}{nq} + h_2 \left(\frac{Dq}{P} + nq \frac{P-D}{2P} \right) - (h_2 - h_1) \left(\frac{qD}{2P} + q \frac{P-D}{nP} \frac{(k+1)k}{2} \right). \tag{24}$$

Once again, by differentiating with respect to q and setting the function obtained equal to zero, the optimal quantity q^* is found to minimize total costs:

$$q^* = \sqrt{\frac{(A_1 + nA_2)(D/n)}{h_2((D/P) + n(P-D)/(2P)) - (h_2 - h_1)((D/2P) + (P-D)/(nP)(((k+1)k)/2))}} \tag{25}$$

and obtaining a minimum cost equal to:

$$C(q^*) = 2\sqrt{\left((A_1 + nA_2) \frac{D}{n} \right) \left(h_2 \left(\frac{D}{P} + n \frac{P-D}{2P} \right) - (h_2 - h_1) \left(\frac{D}{2P} + \frac{P-D}{nP} \frac{(k+1)k}{2} \right) \right)}. \tag{26}$$

Finally, the maximum level of the buyer's stock will be:

$$Mag_{\max}^b = (n-k) \cdot q - (n-k-1) \cdot q \cdot \frac{D}{P} \tag{27}$$

under the obvious condition of $n \geq k$. In particular, it should be highlighted that:

- if $k = 0$, the basic CS model is obtained;
- if $k = n - 1$, the CS-k model matches Hill's approach, i.e. the vendor keeps the entire production in its warehouse and a quantity equal to q is delivered only when the buyer's stock is equal to zero; and
- the total cost may be properly minimized by adjusting n (Peterson and Silver 1979, Hoque and Goyal 2000) for the single-buyer single-vendor situation, with constrained transport capacity.

5.1. *Numerical example*

For the values already assigned, table 1 offers the total costs per year while varying the number of transport operations n and the number of delayed deliveries k. This numerical approach is used in the absence of the analytical model, enabling

	k					
n	0	1	2	3	4	5
1	CS, Hill 2305; 369					
2	CS 2088; 364	CS-1, Hill 2012; 224				
3	CS 2039; 369	CS-1 2003; 267	CS-2, Hill 1929; 164			
4	CS 2035; 376	CS-1 2014; 295	CS-2 1970; 214	CS-3, Hill 1904; 131		
5	CS 2049; 384	CS-1 2035; 316	CS-2 2007; 249	CS-3 1963; 181	CS-4, Hill 1903; 110	
6	CS 2073; 392	CS-1 2063; 333	CS-2 2042; 275	CS-3 2011; 216	CS-4 1969; 157	CS-5, Hill 1915; 096

Table 1. Total cost and maximum level of buyer's stock for a different number of deliveries and delayed supplies.

	Hill	CS-2	CS-1	CS
Optimal production batch size	550	492	474	492
Number of deliveries per batch	5	3	3	4
Maximum level of the vendor stock	352	328	158	123
Maximum level of the buyer stock	110	164	267	376
Total costs per year ($/year)	1903	1929	2003	2035
Set-up costs ($/year)	725	813	844	813
Transport costs ($/year)	227	152	158	203
Vendor holding costs ($/year)	678	554	244	77
Buyer holding costs ($/year)	273	410	757	942

Table 2. Detailed comparison of strategies' performance.

the identification of the number of shipments which minimize the total cost. For the column with $k = 0$, the basic CS model is adopted. Other columns refer to CS-k models.

It is interesting to see how Hill's model results lie on the main diagonal of the matrix. Comparing the detailed results of the four policies (Hill, CS, CS-1 and CS-2), table 2 may be drawn up.

Cases described by CS-k models with $k > 2$ were never best (other than when they coincided with Hill policies) as, for the data given, they did not offer further improvements with respect to the policies mentioned.

Figure 5 shows the behaviour of costs per year as a function of the S level, i.e. the maximum level of the buyer's inventory. It should be highlighted that the non-smooth behaviour of figure 5 curves is a consequence of the non-integer nature of n value.

Of course, Hill's model offers the best result, i.e. the minimum overall cost. However, let us consider the case of a buyer who dedicates a larger space to material stocking, together with a minimum level of material to be maintained, thus accepting a (s, S) range for stock level. In such a case, figure 5 identifies areas of convenience for different CS-k policies. Thus, in response to the question why should a buyer

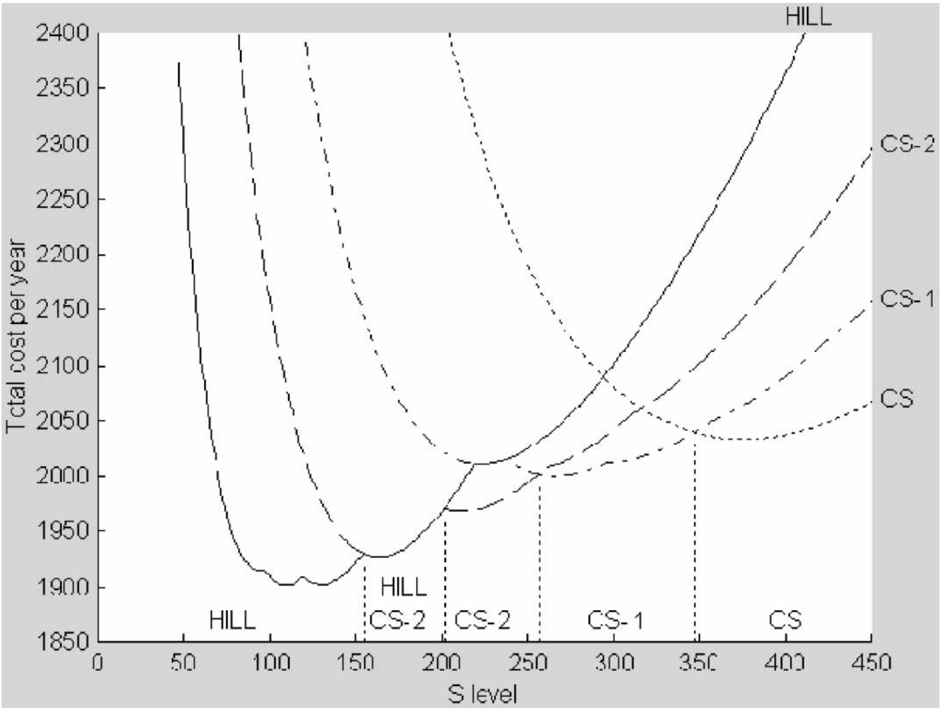

Figure 5. Total system costs for different policies and S levels.

propose and accept such an approach to manage his inventory, the answer is to face demand and/or lead time fluctuations.

6. Stochastic case

To enhance the comparison between the Hill and CS models, a frequent and realistic situation was examined, i.e. the case of stochastic demand. It is plain that Hill's approach offers the lowest costs in a deterministic environment. However, an uncertain environment may modify the situation and the CS approach may prove to be a profitable one.

It is known that demand uncertainties are generally faced by providing safety stocks and so, to compare the two policies, their 'service levels' will be evaluated. To this end, let us define:

- service level S_L as the expected fraction of demand satisfied over the period considered. Of course, quantity $(1 - S_L)$ will be the fraction of demand lost or backlogged; and
- B_{ss} as the number of items in stockout, during the interval between two successive orders (cycle) and given a safety stock equal to ss.

According to Winston (1994), the average amplitude of each stockout is $E(B_{ss})$. As a consequence, the expected stockout per year is $E(B_{ss}) \cdot C_a$, where C_a is the number of cycles in one year, and the following must hold:

$$1 - S_L = \frac{E(B_{ss}) \cdot C_a}{E(D)}, \tag{28}$$

where $E(D)$ is the average demand per year. The expected $E(B_{ss})$ can be evaluated if the distribution of the demand during the lead time (X variable) is known. If it is normally distributed with mean $E(X)$ and standard deviation σ_X, then the safety stock ss $= y{\cdot}\sigma_X$ and it will determine σ_X.NL(y) shortages during the lead time. The values of the normal loss function NL(y) are tabulated (e.g. Peterson and Silver 1979) and, consequently, it is possible to evaluate $E(B_{ss})$ as follows:

$$E(B_{ss}) = \sigma_X.\mathrm{NL}\left(\frac{ss}{\sigma_X}\right). \tag{29}$$

The total costs of the system C_t will be equal to those of the deterministic cases, C_d, plus the safety stock holding costs, i.e.:

$$C_t = C_d + h_2 \cdot ss. \tag{30}$$

It should be emphasized that the CS approach implies the direct control of the buyer's stock by the vendor, i.e. the order emission cost A_2 is lower than in the traditional situation. This fact will be neglected in the remainder of the text, where CS and Hill's model will be compared according to their best performance (i.e. CS-3 with $n > 4$ so as not to have a Hill policy).

7. Case of stochastic demand

According to the parameter values previously assigned, let us consider:

- stochastic demand described by a normal distribution with mean $E(D)$ equal to 1000 (pieces/year) and a standard deviation σ_D between 0 (deterministic case) and 44.72 (pieces/year) (i.e. variance equal to 2000); and
- delivery lead time equal to zero.

With reference to the first point above, the σ_D adopted are undoubtedly low with respect to the mean. However, they are sufficient to show the CS performance even in a situation where the approximation of a sufficiently regular demand to a deterministic one may be considered as a reasonable assumption and, consequently, Hill's hypotheses may apply to the case. Figure 6 shows the levels of the vendor and buyer stocks during the production of a batch. The number of deliveries per batch n is equal to five, thus obtaining a minimum cost also for the CS policy.

The graph also plots the minimum and maximum level that the buyer's stock may reach because of demand variability. When adopting the CS approach, it is evident that the stockout probability is relevant only for the first delivery, as the stock level is sufficiently high in the rest of the cycle (period of time between the production of two consecutive batches). It should be noted that the delivery lead-time is null, but the batch is to be produced, so that there exists a 'system lead time' other than zero. The system lead-time $\mathrm{lt_s}$ is equal to $\mathrm{lt_s} = q/P$ and the number of cycles C_y in a year is $C_y = E(D)/(n \cdot q)$. In the case described, $\mathrm{lt_s} = 0.0334$ (years), i.e. 12 (days), and $C_y = 1.87$ (cycles/year). The standard deviation of demand during the $\mathrm{lt_s}$ interval is:

$$\sigma_X = \sqrt{\sigma_D^2 \frac{q}{P}} = \sigma_D \sqrt{\frac{q}{P}}\,. \tag{31}$$

It is also possible to evaluate the behaviour of Hill's model for a normally distributed demand (figure 7).

The same figure 7 refers to Hill's model optimal situation (batch equal to 550 (units) and five deliveries per batch): because of demand fluctuation, the arrival of

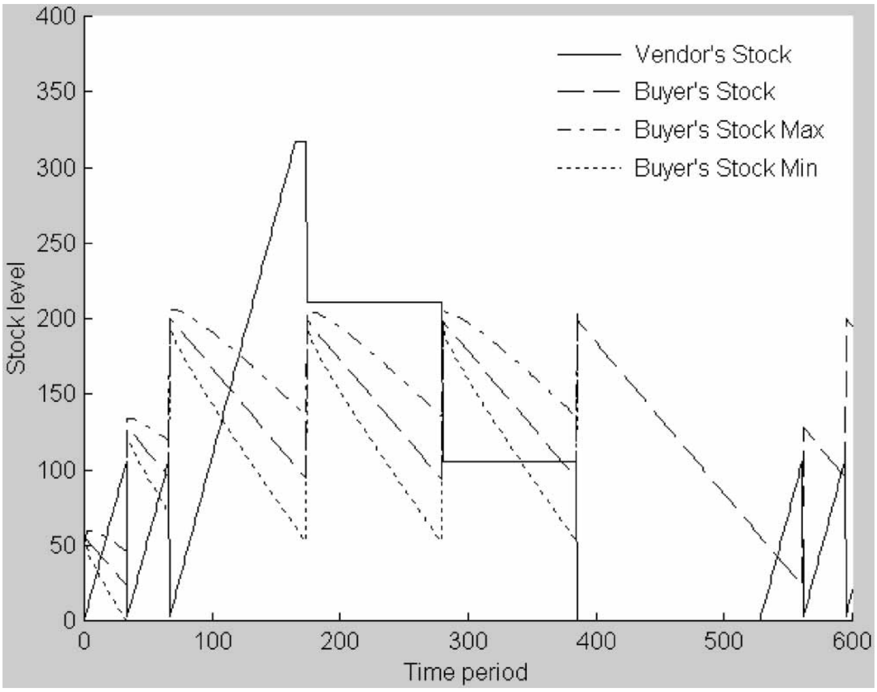

Figure 6. Buyer's and vendor's stocks for a stochastic demand and CS-3 policy.

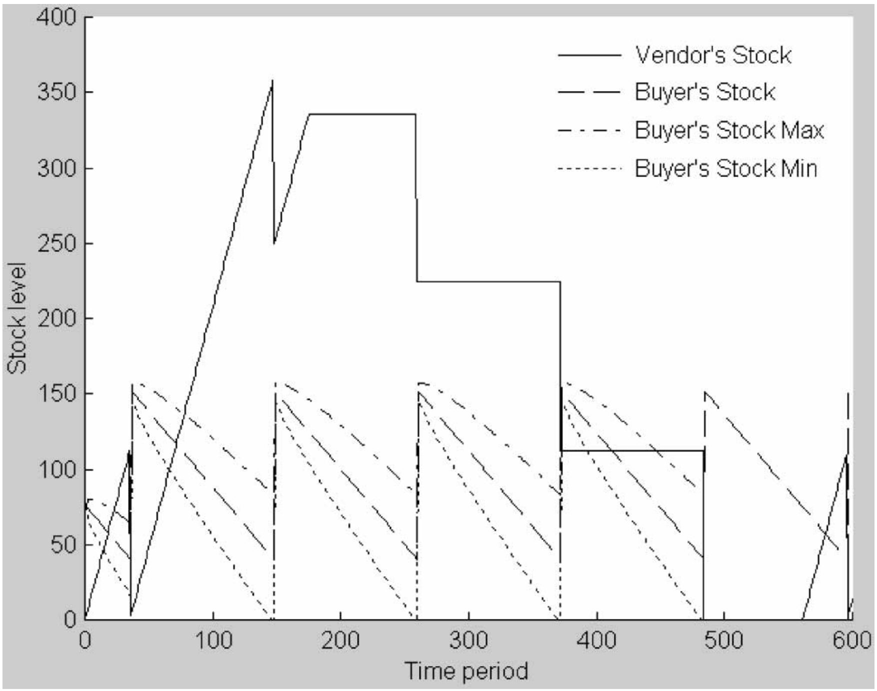

Figure 7. Hill's model and normally distributed demand.

each delivery is a critical situation, as a stockout may occur. In such a case, the system lead-time lt_s is the lapse of time between two consecutive deliveries, i.e. $lt_s = q/E(D) = 40$ (days), and $C_y = E(D)/q = 9.09$ (cycles/year).

The standard deviation of demand during the system lead-time is:

$$\sigma_X = \sqrt{\frac{\sigma_D^2 \cdot q}{E(D)}} = \sigma_D \sqrt{\frac{q}{E(D)}} \tag{32}$$

and it is possible to calculate the safety stock required when adopting Hill's model.

When figures 6 and 7 are compared, it also emerges that:

- in Hill's model (figure 7), the safety stock is constantly required during each period, because of the saw-tooth aspect of the buyer's stock; and
- in the CS approach (figure 6), the safety stock is really necessary only during the first deliveries, i.e. when the buyer's stock is at its lowest levels. Nevertheless, in the following, the safety stock will be considered as applied during each period. Its value may be regarded as the starting basis for the *s* level bargaining activity implicit in a CS agreement.

7.1. *Numerical example*

Let us assume a service level $S_L = 99.98\%$ and a delivery lead time equal to zero. S_L has been set to an unrealistically high value to emphasize the CS performance, given the data set assumed from Goyal's example. However, the same effect could be obtained with the more frequent case of the combination of a lower service level and higher demand variability. Formulae proposed in section 6 offer the results shown in figure 8.

As the assigned standard deviation increases, Hill's model requires an increased safety stock, with respect to the CS approach, to guarantee the service level S_L. Thus,

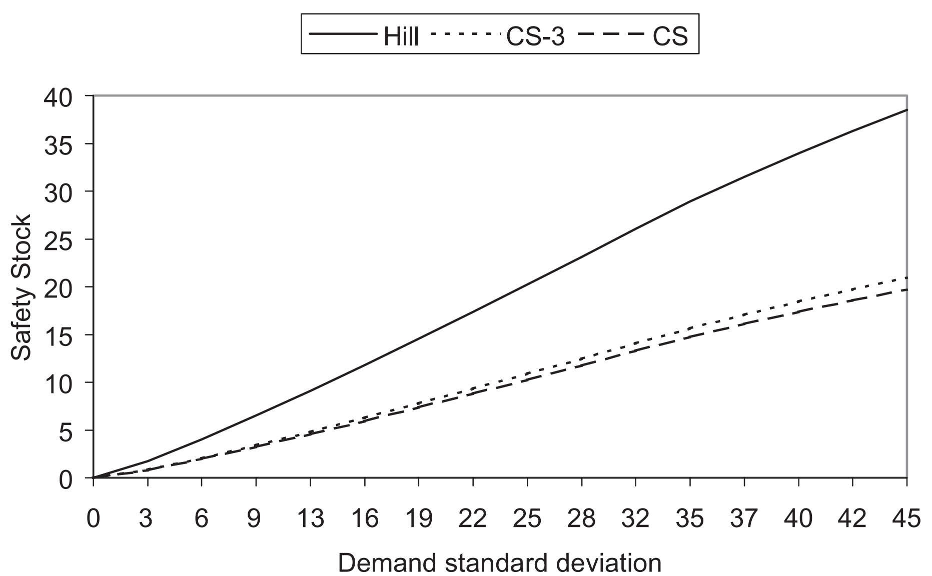

Figure 8. Safety stocks for different models and demand standard deviations.

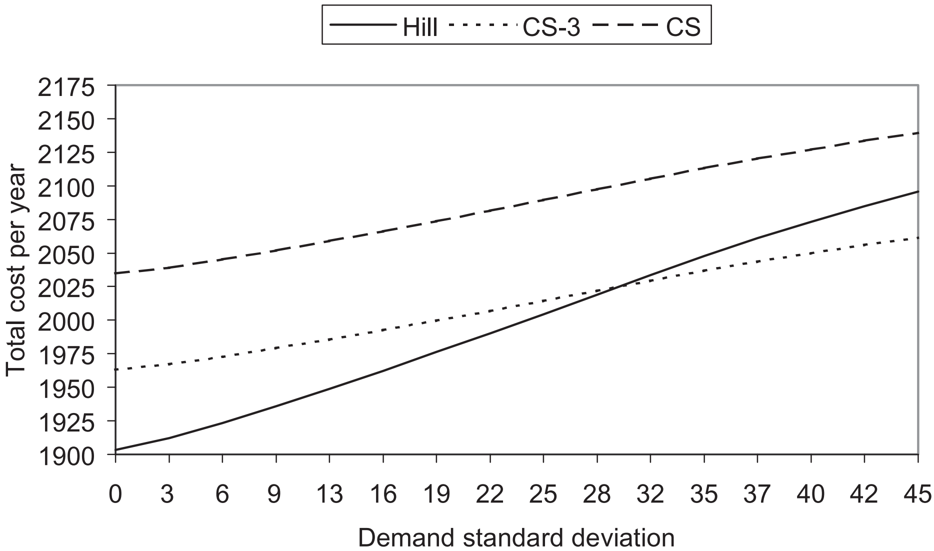

Figure 9. System costs, including safety stocks, versus demand standard deviation.

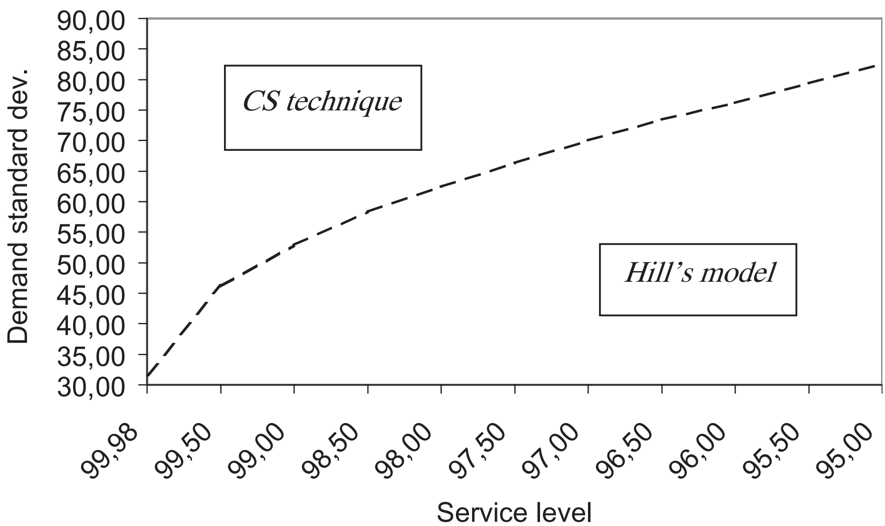

Figure 10. Areas of convenience for variable demand, standard deviation and service level.

total costs rise (figure 9). Figure 9 shows how, for a demand standard deviation greater than 30, the CS-3 model offers lower costs than Hill's model. The results obtained have been verified by simulation experiments, which are not reproduced for the sake of brevity.

Safety stocks may also be calculated for different service levels and demand standard deviation σ_D: for an assigned S_L there exists a σ_D (σ_{limit}) so that Hill's model is to be preferred to CS when $\sigma_D < \sigma_{\text{limit}}$. Figure 10 summarizes the whole set of results obtained, thus showing a borderline that distinguishes the area of Hill's model convenience from the CS area of outperformance.

8. Conclusions

Starting from an industrial practice observed in a manufacturing company, the present study described a policy for the management of stocks in a Supply Chain, named Consignment Stock (CS). To evaluate its performance, an analytical model was developed and a comparison made with Hill's model. The results obtained helped in understanding the CS mechanism, also offering a procedure for identifying those situations where it could be adopted successfully. Further work on the subject might help in the complete understanding of the CS potential. In particular, investigations are in course to evaluate the proper s and S levels and to examine the cases of multibuyer and multivendor environments.

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