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Model and analysis of integrated production–inventory system: The case of steel production

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Abstract

The study originated from an industrial case study in the field of steel production, but it presents a larger interest, as many other manufacturing fields have similar concerns (e.g. foundries, food, textile and paper industries). A significant phase of steel manufacturing is the product cooling (likewise, drying in paper and textile production, or maturing in food production). This phase may be completed in different ways, but (1) it must be carried out in the finished product warehouse and (2) it must meet both production optimisation and customer needs. The latter requirement acquires a strategic relevance in JIT environments. The present study proposes a mathematical model to find the optimal production schedule of steel billets, based on the relevant parameters of the productive system (set-up and processing times, demand profile). In the industrial case examined, the negative impact of holding costs on cash flows is also linked to the space required by the cooling process, which depends on the production schedule adopted. In other words, the finished product storage can be considered a part of the manufacturing cycle and impacts on it. In the case of steel plants operating in JIT environments, the warehouse must be promptly emptied and carefully managed to exploit the available space. Thus, the effect of inventory costs is examined in a production–inventory system with finite capacity, where products are made to order and share the same manufacturing facility. The study is completed by an experimental analysis to investigate the effect of variations in the relevant parameters of the problem.

Keywords: Capacitated production–inventory system; Scheduling; Linear programming; Continuous casting

1. Introduction

This case examined here refers to the production planning of a "mini steel plant", where the continuous increase in productivity led to problems of space in the final product warehouse. In this area, billets are cooled before customer withdrawal. The final aim of the present work is to establish the optimal production sequence of the billets, ordered by the customers, while taking into account the limited space available in the warehouse.

The utilisation of mathematical methods for production optimisation in steel plants is not new.

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Tibor (1958) introduced a linear mathematical model to optimise the mix of raw materials and energy consumption. Subsequently, several contributions involving mathematical programming were published on the same problem. Nevertheless, existing research does not consider in detail the problem of the mix of raw materials: it is not a salient feature of mini-steel plants with electrical furnace, as the charge is composed by iron scraps and alloy elements. This study focuses on the optimisation of the production schedule, i.e. the billet type to be produced and in period of the time horizon. A recent survey on this topic may be found in Tang et al. (2001), who show how proposed models refer to the optimisation of the typical stages of steel production, i.e. steel making (SM), continuous casting (CC) and hot rolling (HR). The optimisation may be carried out for the whole system or a part of it. Our contribution is focused on the CC phase, a unique phase of mini-steel plants (the final products are billets, differing in quality, section and length). The billet warehouse will be considered as a part of the productive cycle, thus introducing an original problem of industrial relevance.

According to Tang et al. (2001), the following approaches were adopted for production planning in steel plants:

- OR (Operational Research);
- AI (Artificial Intelligence):
  - Expert Systems;
  - Intelligent search methods, such as GA (Genetic Algorithm), SA (Simulated Annealing) and TS (Tabu Search);
  - Constraint satisfaction;
- Human–machine coordination methods
- Multi Agent methods

In order to solve the problem, a mixed integer linear model will be proposed (Section 5). In this regard, Lally et al. (1987) introduced a mixed integer linear model for the solution of the billet scheduling problem in CC. They consider a simplified model for a steel plant. Mohanty and Singh (1992) proposed a hierarchical system with two levels for production planning. The problem is formulated as a goal-programming model, solved by a multi-objective dynamic algorithm. The result is an aggregate plan guaranteeing the best use of the available resources. Chen and Wang's (1997) linear model is developed in the Supply Chain perspective, thus looking for the optimal production plan in a given context of semi-finished or raw material supply and finished product distribution. However, the optimal solution of the problem is found with a reduced number of variables and heuristics are not presented for a more realistic solution. In Kalagnanam et al. (2000), the influence of a billet warehouse is introduced into the production planning of the CC process. Firstly, demand is satisfied by surplus billets stocked in the warehouse (if any) and, subsequently, by scheduling the remaining orders. The problem is formulated as a bicriteria multiple knapsack model with additional constraints, solved with a heuristic. In Kapusinski and Tayur (1998), a model for production optimisation is proposed, where the relationship between the system capacity (and, therefore, the warehouse) and the cost/profit deriving from its utilisation is introduced and discussed. Finally, Tang et al. (2002) enhance their previous model (Tang et al., 2000) by a linear mixed integer model. The integrated optimisation of SM and CC production considers multiple units available in each of two stages. The model proposed is solved by a heuristic based on Lagrangian relaxation.

2. The manufacturing process

The CC technology was developed in the 1970s. Today, it is widely used by Italian steel makers. The lack of iron mines prompted the development of production processes based on the ferrous scrap fusion by Electric Arc Furnaces (EAF). The billet CC process starts with the introduction of scraps into the EAF. Then, Ferro-alloys are added and degassing is carried out by the injection of argon or nitrogen at the Ladle Furnace. At the end of the fusion process, molten steel is moved in the ladle and, from it, to the tundish, where the metal-static head is kept constant to guarantee a regular outflow in the operating lines (the steel plant considered has five parallel lines). A bottomless copper container is placed under the tundish and cooled by demineralised water circulation (mold). The formation of a thin solid layer allows the metal to sustain itself. At the mold exit, the billet is cooled by a jet of water before cutting. The CC machine operates continuously, without interruption, until the ladle is completely emptied. It must be replaced promptly, with a full one, so that production is managed with a minimum number of interruptions. A schematic representation of the CC process is shown in Fig. 1.

The production process can be summarised as follows: (1) preparation of the iron scraps and alloy elements (EAF load); (2) fusion; (3) CC of the billets, (4) billet cooling. Billet length ranges from 1.6 to 12m; sections are square, with sides of 100, 115, 120, 130, 140 and 160 mm. Different qualities of steel are produced (approximately, 100 types). Production is continuous, using three shifts per day over the whole week, for a total of 330 working days per year. The EAF capacity is equal to 65 tons, with a 6200 mm diameter. The maximum production is estimated to be 1600–1700 tons per day, i.e. a potential output of about 600,000 tons per year.

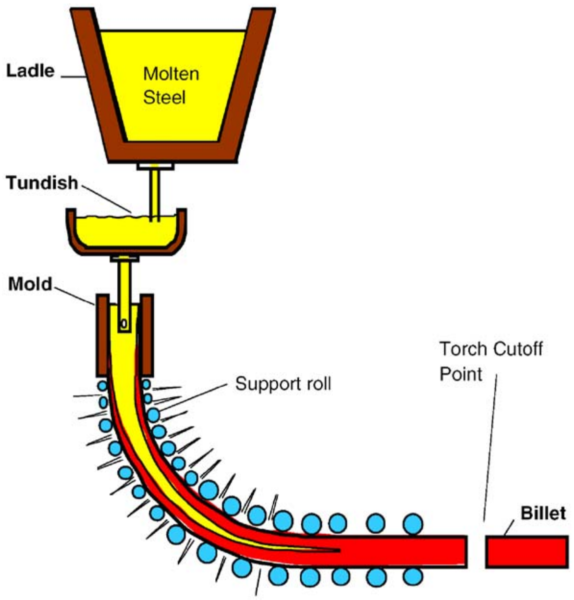


Fig. 1. Continuous billet casting process.

3. The billet warehouse

The warehouse for billet storing and cooling is a $57 \times 26\,m^2$ area, where an overhead crane equipped with an electromagnet transports the products. The warehouse size, further reduced because of billet handling, represents an important constraint (billets cannot be stocked anywhere else). Therefore, the initial study focussed on the possible effects of enlarging the warehouse. At present, if customer orders are small (i.e., few casts), billets are arranged in a single layer, thus cooling rapidly (<12 hours) to allow truck or railway transportation. If demand is large (i.e., several casts), billets are stocked in multiple layers and cooling times increase (roughly, over 12 hours for each layer added). Of course, there exists a limit to the maximum number of layers, depending on the length and side of the billets (the absolute maximum is 13 layers, for billets 115 mm side and 2 m long).

4. The industrial context

The analysis of production data (years 2000–2001) showed how billet orders range from a few to dozens of thousands of tons. Customers were classified according to the tons ordered per year, thus obtaining the ABC graph in Fig. 2. The graph shows how one customer accounts for almost half the total demand and several customers order smaller quantities of steel.

The distribution of the order size is shown in Fig. 3 (wide intervals are given due to industrial reserve).

It should be highlighted that the company produces a wide range of different steel types. Their quality depends on the chemical composition (i.e. C, Mn, Si, P, S, Cu, Cr, Ni, Mo, Sn and Al content) and production scheduling is influenced by the compatibility of the alloy produced. Therefore, steel qualities were categorised into nine homogeneous groups (Fig. 4), each containing those materials which can be produced consecutively without quality problems.

If the schedule implies the consecutive production of steel belonging to incompatible groups,

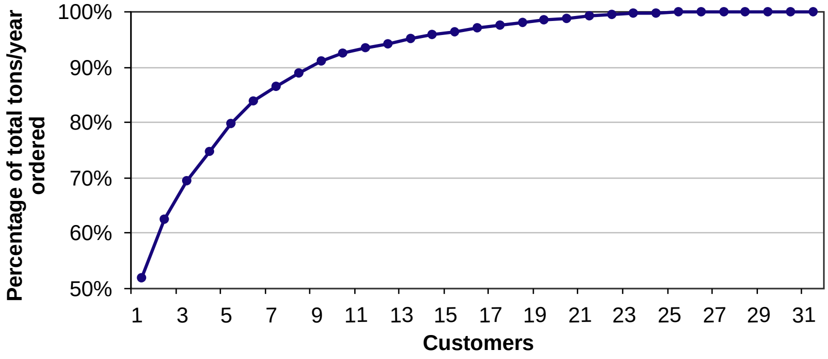
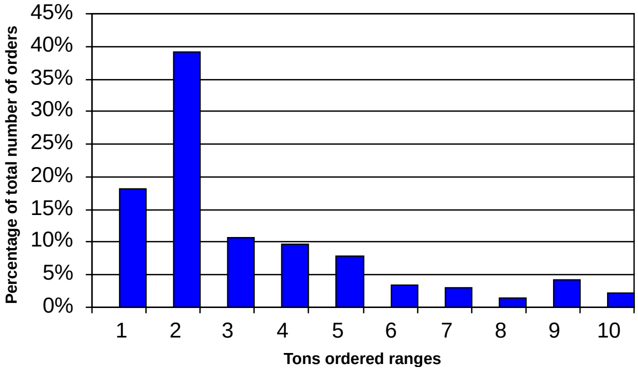


Fig. 2. ABC analysis of customers.



Ranges	
n°	[tons]
1	<100
2	100-500
3	500-1000
4	1000-1500
5	1500-2000
6	2000-3000
7	3000-4000
8	4000-5000
9	5000-10000
10	>10000

Fig. 3. Order size.

Group	1	2	3	4	5	6	7	8	9
1	1	2	2	NC	NC	NC	NC	3	NC
2	2	1	2	NC	NC	NC	NC	NC	NC
3	2	2	1	2	3	NC	NC	NC	2
4	NC	NC	2	1	NC	3	NC	NC	2
5	NC	NC	3	3	1	3	NC	NC	3
6	NC	NC	NC	3	3	1	NC	NC	NC
7	NC	NC	NC	NC	NC	NC	1	NC	NC
8	3	NC	NC	NC	NC	NC	NC	1	NC
9	NC	NC	2	2	3	NC	NC	NC	1

Legend	
1	High compatible
2	Compatible
3	Low compatible
NC	Incompatible

Fig. 4. Compatibility between steel groups.

there is a need to degrade the first billets of the new casting. This practice is necessary because of the presence of alloy elements in the CC system during the transition period between two castings. Therefore, the casting sequence of two incompatible groups implies that about 10 tons of billets are sold at 30% lower price, thus resulting in a profit loss.

5. Mathematical formulation of the problem

Linear programming (LP) is a widely used technique of mathematical programming and several applications to real-world problems are quoted in the literature. Our first aim was to develop a linear model, even if this may introduce some limitations in to the detailed formulation of the problem, it allows efficient solution by standard linear programming software packages. The notation adopted is shown below:

- i billet type index ($i = 1, \ldots, n$);
- j index of the planning horizon day ($j = 1, \ldots, m$);
- h horizon of production planning (28 days);
- l_i length of the ith billet (from 1.6 to 6 m);
- s_i side of the square section of the ith billet (115, 120, 130, 140, 160 mm);
- NBC_i number of the ith billet produced for each cast (maximum capacity);
- layer_i number of layers of the ith billet stack;
- Pt production time for each cast (40 minutes);
- Hd hours available for production (24 hours/day for 330 days/year);
- Ic average holding cost (referred to the cost of one cast stocked for 1 h in the warehouse);
- Pr average profit of one cast;
- St average time for the CC process set-up;
- O_{ij} quantity (casts) of the ith billet ordered for the jth day;
- L_{Inv} warehouse length (57 m);
- H_{Inv} warehouse width (26 m);
- Io_i starting inventory of the ith billet;
- cool_{ik} cooling time (days) of the ith billets (minimum time required between production and transport to the customer). This value depends on the number of possible levels (layer_i) for each ith billet type;
- M large number (larger than the maximum number of casts that can be produced per day);
- aux_i auxiliary variable for calculating the space occupied by a cast of the ith billet: $\mathrm{aux}_i = l_i/(H_{\mathrm{Inv}} \cdot \mathrm{layer}_i)$;
- C_s penalty cost due to orders, scheduled for the sth week but fulfilled later ($s = 1, 2, 3, 4$), i.e. assuming the possibility of lateness in delivery;
- P_{ij} variable; number of casts of the ith billet produced in the jth day;
- I_{ij} variable; number of casts of the ith billet in the warehouse during the jth day;
- S_{ij} variable; number of casts of the ith billet delivered to the customer in the jth day;
- y_{ij} binary variable; set-up of the EAF for the production of the ith billet type in the jth day

$$y_{ij} = \begin{cases} 1 & \text{if billet } i \text{ is produced in day } j, \\ 0 & \text{if billet } i \text{ is not produced in day } j. \end{cases}$$

The model

Objective function:

$$\mathrm{MAX}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}[\mathrm{Pr} \cdot S_{ij} - I_{ij} \cdot \mathrm{Ic} \cdot \mathrm{Hd}] - C_1 \cdot \sum_{i=1}^{n}\sum_{j=1}^{h/4}(O_{ij} - S_{ij}) - C_2 \cdot \sum_{i=1}^{n}\sum_{j=h/4+1}^{h/2}(O_{ij} - S_{ij}) - C_3 \cdot \sum_{i=1}^{n}\sum_{j=h/2+1}^{3 \cdot h/2}(O_{ij} - S_{ij}) - C_4 \cdot \sum_{i=1}^{n}\sum_{j=3 \cdot h/4+1}^{h}(O_{ij} - S_{ij})\right].$$

Constraints:

$$\sum_{i=1}^{n}[\mathrm{Pt} \cdot P_{ij} + \mathrm{St} \cdot y_{ij}] \leqslant \mathrm{Hd} \quad \forall\, j = 1, \ldots, m, \tag{1}$$

$$I_{i1} = \mathrm{Io}_i + P_{i1} - S_{i1} \quad \forall\, i = 1, \ldots, n, \tag{2}$$

$$I_{ij} = I_{i,j-1} + P_{ij} - S_{ij} \quad \forall\, i = 1, \ldots, n,\ j = 2, \ldots, m, \tag{3}$$

$$\sum_{j=1}^{h} P_{ij} = \sum_{j=1}^{h}(O_{ij} - \mathrm{Io}_i) \quad \forall\, i = 1, \ldots, n, \tag{4}$$

$$\sum_{j=1}^{h} S_{ij} = \sum_{j=1}^{h} O_{ij} \quad \forall\, i = 1, \ldots, n, \tag{5}$$

$$\sum_{d<j} S_{id} \leqslant \sum_{d<j} O_{id}, \quad \forall\, i = 1, \ldots, n, \; j = 2, \ldots, m, \tag{6}$$

$$S_{i,j+\mathrm{cool}_{ik}} \leqslant P_{ij}, \quad \forall\, i = 1, \ldots, n, \; j = 1, \ldots, m, \tag{7}$$

$$y_{ij} \geqslant \frac{P_{ij}}{M} \quad \forall\, i = 1, \ldots, n, \; j = 1, \ldots, m, \tag{8}$$

$$\sum_{i=1}^{n} I_{ij} \cdot s_i \cdot \mathrm{NBC}_i \cdot \mathrm{aux}_i \leqslant L_{Inv} \quad \forall\, j = 1, \ldots, m. \tag{9}$$

The objective function aims to maximise the profit of billet sales, taking into account the holding costs. Therefore, indirectly, it aims to minimise the billet cooling time. The objective function contains four additional terms accounting for the orders unsold within the assigned week (delayed delivery). In particular, a decreasing penalty cost ($C_1 > C_2 > C_3 > C_4$) is assumed from the first to the fourth week of the time horizon.

Basically, three cost types are introduced.

- *Holding costs*: they are directly proportional to the time interval T during which a billet remains in the warehouse, even if it is still cooling;
- *Production costs*, which are classified as follows:
  - the cost of raw materials;
  - the energy cost, which depends on the quality of the iron scraps used and on the quality of the steel;
  - the cost of direct and indirect labour;
  - the cost of ordinary and extra-ordinary maintenance.
- *Costs due to delayed orders*, as already mentioned.

It should be emphasised that costs are coherent with the case observed, although they are not presented due to company confidentiality needs. The constraint set (1) represents the productive time available per day (set-up plus production times must be less than 24 hours). The quantity of material in the warehouse, per type and day, is taken into account by (2) and (3) constraints. Constraint (4) imposes a limit on the production in the planning period (equal to the sum of the orders in the same period minus the number of billets initially available in the warehouse). In the planning horizon, the equality between sold and ordered billets is imposed (constraint (5)), whereas constraint (6) guarantees that the quantity of sold billets up to the dth day ($d = 1, \ldots, H$) is smaller or equal to the quantity to be delivered (deliveries cannot be anticipated). The sets of constraints (7) and (8) are introduced to ensure a linear model. The former implies that the sales of the ith product in the jth day correspond to the billet production in the ($j - \mathrm{cool}_i$) day. The latter is introduced to assign the value of the Boolean variable y_{ij}. Finally, the last constraint (9) imposes a limit on the warehouse capacity: billets stored for cooling cannot exceed the space available. Consequently, the production of billets will be limited by this constraint.

6. Implementation and solution of the model

A VBA programme was developed to generate orders coherent with the demand profile of the company: they will represent the input for the model. In the specific industrial case considered, the linear model proposed (Section 5) requires 8960 variables and 9176 constraints. It was implemented in the MPL language and solved by a CPLEX 6.6.0 MIP optimiser. The output gives:

1. billet type (and quantity) to be produced for each day of the planning horizon;
2. completion date for each order;
3. number of days required to complete each billet order;
4. quantity of billets stored in the warehouse;
5. production to be sold per day;
6. final profit.

The planning horizon (4 weeks) may be easily modified. As usual, computational time is a strategic parameter to be considered in evaluating the performance of the model. Simple cases are solved in a few seconds. On the other hand, if the set of orders uses up most of the available capacity, the solution procedure may find difficulty in quickly identifying a feasible solution, thus the computational effort may take several hours.

It should be emphasised that the number of stack layers (indirectly, the cooling time of stacks) should be a decisional variable. Such a formulation adds to the model complexity and the consequent computational time. The results of various experiments are presented to show the behaviour of the model and to introduce an efficient solving heuristic.

7. Computational tests

Different demand profiles were generated randomly, according to the industrial situation examined (Section 4). At first, the profit trend was investigated while varying the warehouse size (Fig. 5). Up to certain value, an increasing relationship between the dimension of the warehouse and the profit is observed. Beyond this, a larger warehouse does not add any benefit. On the other hand, Fig. 6 shows how specific demand profiles could require a larger warehouse to suit alternative production schedules that are able to further increase the profit generated.

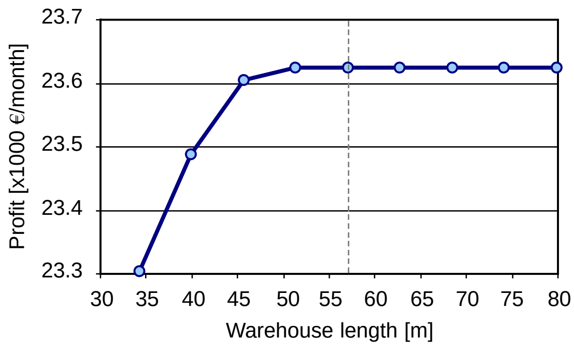


Fig. 5. Trend of the profits with varying warehouse length.

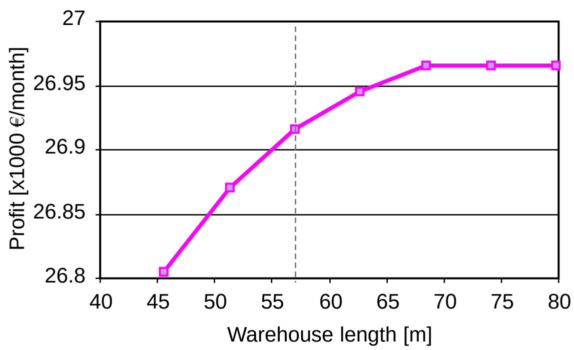


Fig. 6. Trend of the profits with varying warehouse length.

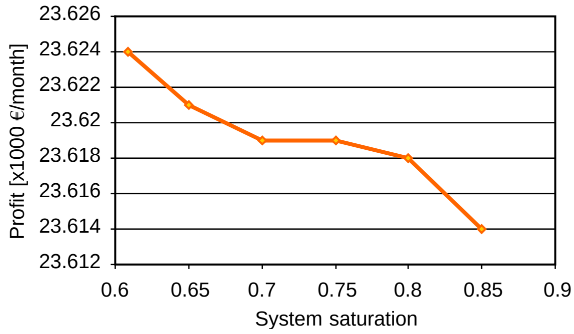


Fig. 7. Profit trend with varying system saturation.

A second set of computational tests (Fig. 7) studied the profit trend with respect to system saturation (i.e. the minimum utilisation of the EAF). According to a given demand profile, profit may decrease because of the need for a production amount larger than demand. The EAF saturation plays a significant role in steel plants, as production activity must be maintained to prevent the costs resulting from system interruptions. The EAF utilisation cannot fall below a threshold, thereby determining the poor utilisation of some components (e.g. electrodes and refractories). Therefore, make-to-stock production may be an unavoidable choice in steel plants, which is also discussed in Kalagnanam et al. (2000).

The third set of computational tests investigated the profit trend while varying the number of layers in the billet stacks. A positive coefficient α was adopted to give the number of stack layers for the ith billet, i.e. $\lceil \alpha \cdot \max(\text{layer}_i) \rceil$ with $0 \leqslant \alpha \leqslant 1$. Fig. 8 shows the trend of the two situations related to the introduction of the warehouse capacity constraint. The profit data show a monotonic trend, i.e. lower the α coefficient, the higher the profit. Obviously, this fact can be explained by the lower holding costs (due to the reduced cooling times) and the prompt delivery to the customers.

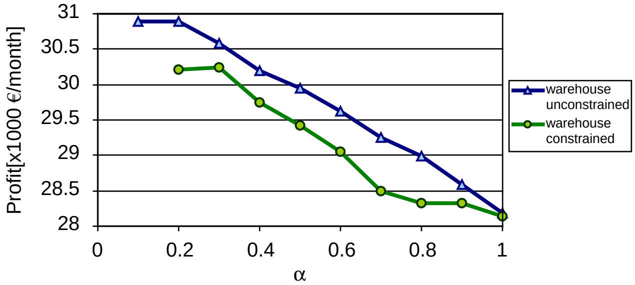


Fig. 8. Trend of the profits at varying of the number of stack layers.

8. The algorithm

The model in Section 5 offered acceptable results for a significant part of the problems considered. However, the optimal solution cannot be identified for the given situation, e.g. when the order plan determines the EAF saturation and prompt delivery is required (low number of layers). To overcome this problem, different optimisation algorithms were compared and the best performing one is here, as follows:

Initial scheduling search:

1. Compute the optimal production plan with $\alpha = 0.1$
   (a) If solution is feasible → optimal solution → end
   (b) Otherwise → go to step 2
2. Compute the optimal production plan with $\alpha = 1$
   (a) If solution is feasible → go to step 3
   (b) Otherwise → the problem is unfeasible, check input data.

First schedule improvement:

3. Relax the warehouse capacity constraint and compute the optimal production plan with $\alpha = 0.1$, set $h{:=}0$; Calculate $\mathrm{Space}_{ij} = I_{ij} \cdot s_j \cdot \mathrm{NBC}_i$ according to the production plan just generated → go to step 3(a)
   (a) Considering all the ith products, complete set I' with the elements for which $\max_j \mathrm{Space}_{ij} \geqslant L_{\mathrm{Inv}}$; then, set variable $\mathrm{layer}_i = \max(\mathrm{layer}_i)$ $\forall i \in I'$ → go to step 4.
   (b) Considering all the jth days, complete set I''_j with the product elements for which $\sum_i \mathrm{Space}[i,j] \geqslant L_{\mathrm{Inv}}$; according to Space_{ij} order the products $i \in I''_j$ increasingly, then, set variable layer_i at value $\max(\mathrm{layer}_i)$ for i in hth position → go to step 4.
4. Compute the optimal production plan considering the warehouse capacity constraints and the variable layer_i
   (a) If solution is feasible → go to step 5
   (b) Otherwise → $h{:=}h+1$ and go to step 3(b)

Second schedule improvement:

5. Starting from production schedule (4(a)) and the related $\mathrm{Space}_{ij} = I_{ij} \cdot s_j \cdot \mathrm{NBC}_i \cdot \mathrm{aux}_i$ values, go to step 5(a)
   (a) $\forall i \in [I', I''_j]$ with $\frac{1}{4}L_{\mathrm{Inv}} \leqslant \max_j \mathrm{Space} \leqslant \frac{1}{2}L_{\mathrm{Inv}}$ set the variable $\mathrm{layer}_i = \lceil \frac{1}{2}\max(\mathrm{layer}_i) \rceil$ Compute the optimal production plan with warehouse capacity constraints and new variable layer_i
      1. If solution is feasible → go to step 5(b)
      2. Otherwise → keep point 4 solution → end
   (b) $\forall i \in [I', I''_j]$ with $\frac{1}{8}L_{\mathrm{Inv}} \leqslant \max_j \mathrm{Space} \leqslant \frac{1}{4}L_{\mathrm{Inv}}$ set the variable layer_i at value layer_i at value $\lceil \frac{1}{4}\max(\mathrm{layer}_i) \rceil$ Compute the optimal production plan with warehouse capacity constraints and new variable layer_i
      1. If solution is feasible → go to step 5(c)
      2. Otherwise → keep point 5(a) solution → end
   (c) $\forall i \in [I', I''_j]$ with $\max_j \mathrm{Space}_{ij} \leqslant \frac{1}{8}L_{\mathrm{Inv}}$ set the variable layer_i at value layer_i at value $\lceil \frac{1}{8}\max(\mathrm{layer}_i) \rceil$ Compute

the optimal production plan with warehouse capacity constraints and new variable $layer_i$

1. If solution is feasible $\rightarrow$ keep point 5(c) solution $\rightarrow$ end.
2. Otherwise $\rightarrow$ keep point 5(b) solution $\rightarrow$ end.

The algorithm proposed above was successfully applied in several situations. As an example, a production plan consisting of 700 casts was found feasible only with $\alpha = 1$. The first part of the algorithm led to a 3.57% improvement in the objective function, while the second part resulted in an additional 7.28% improvement. Also significant computational time savings result from the utilisation of the algorithm proposed. These results pertain to specific demand profiles, but experimental tests were carried out to validate the algorithm definitively.

9. Conclusions

This study proposed a mathematical model for production planning in the CC process of a mini steel plant. The software developed allows to optimise the production schedule automatically. Starting from the industrial case, the model considers the billet-cooling area (warehouse) as an integral part of the productive system. This practical aspect renders the present work original and of considerable industrial significance. The study is completed by a set of computational tests to validate the model itself; taking into account the possible variations of the relevant parameters of the productive system. The results obtained show how the constraint introduced by the warehouse capacity impacts on the production schedule. Therefore, an algorithmic procedure is proposed in order to identify infeasible demand profiles and generate a ‘‘good’’ production schedule, when the optimal solution cannot be found. Further computational experiments are performed to validate the optimisation algorithm.

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