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Planned lead time determination in a make-to-order remanufacturing system ☆

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Abstract

In recent years, remanufacturing has emerged as an important research area, due to the tendency of stricter environmental regulations in industry and the awakening to the economic attraction of recovering the products rather than the disposal alternative. This also requires developing manufacturing planning and control techniques to improve the performance of remanufacturing systems.

In order to reassemble finished products, new components are required since the recovery rate of return components can never reach 100%. When making a disassembly and procurement decision, we then need to balance the inventory holding cost and stockout cost. In the meantime, the process lead time depends on which disassembly and procurement option that is chosen. In this paper, we study a system where remanufacturing is driven by customer orders. A disassembly order is always released first and then the disassembly result determines whether a purchasing order is needed. Our objective is to examine the process lead time, which can be used to determine the planned lead time in production planning and control of remanufacturing. We start with disassembling a single-component case and then extend the model to a two-component scenario. We also investigate how the disassembly yield influences the system performance. Results of this study are intended to be implemented in a real-world engine remanufacturing environment.

Keywords: Disassembly; Remanufacturing; Reverse logistics systems

1. Introduction

The reuse field has grown considerably in the past decades, due to its economical benefits and environmental requirements. Remanufacturing, representing a higher form of reuse and focusing on value-added recovery, has been introduced in many fields, such as in the automotive, telecommunication, electrical equipment and machinery areas. In addition to the economic profitability, there is an increasing number of legislation restrictions that assign to the producers the responsibility for taking care of used products, for instance, EU Directive 2002/96/EC and 2003/108/EC related to the *Waste Electrical and Electronic Equipment* and EU Directive 2002/525/EC related to the *End of Life Vehicles*. Remanufacturing has become an important industrial sector to achieve the goal of sustainable development.

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A large volume of literature in the reuse filed has emerged in recent years. Gungor and Gupta (1999) take a comprehensive survey and classify the published work into different categories. Regarding product and material recovery, there are several issues being addressed in literature, namely collection, disassembly, inventory control and production planning. Collection deals with the reverse logistics system which aims at collecting the used products and package efficiently. Disassembly often addresses the disassembly strategy (how far to disassemble?) and disassembly process planning (in which way to disassemble?) questions. Among others, Penev and de Ron (1996), Johnson and Wang (1995, 1998) present models to decide the disassembly sequence and routing, with the aim of minimising the operational costs while fulfilling the production due date. A recent survey study in disassembly is also given by Lambert (2003).

A wide range of models have been developed with the aim of improving inventory control in product recovery process, for instance, the PUSH and PULL policies investigated by van der Laan (1999) and Teunter et al. (2002). Several other policies are also available differing in terms of the definition of inventory position (Kiesmüller 2003). Other similar approaches, related to base stock policies, can be found in Vlachos and Dekker (2000) in which the aim is to determine single period order quantities for an individual product, and in Kiesmüller and Minner (2003), which extends the determination of optimal order quantities to the multi-period case with significant recovery and production lead times. Nevertheless, the above models are mainly ‘‘base stock’’ type of inventory control systems.

Production planning systems are often used in enterprises with more complicated products such as engines. In this case a product recovery process can be roughly divided into three stages: Disassembly, remanufacturing and reassembly. Disassembly is the first step in product recovery and acts as an information gateway for production planning (Guide, 2000). Return products are disassembled, assessed and then purchasing requirements are generated in order that sufficient new parts are procured. Production planning and control activities become more complicated because of multi-dimensional material flows, uncertainties of the conditions of the returned products, and imbalances in return and demand rates, etc. (Guide, 2000). Managing such a production process is a very challenging job in practice, due to the lack of specific technology and planning and control systems for remanufacturing.

In recent literature concerning production planning and control of remanufacturing systems, models are often developed for fast moving and standard items. Regarding the make-to-order (MTO) remanufacturing environment, limited efforts appear to have been devoted. According to Guide (2000), coordinating disassembly, remanufacturing and reassembly processes is essential to satisfy material match requirements. This sometimes leads to an MTO production strategy, with a cost usually 25% higher than in a make-to-stock (MTS) situation. Guide et al. (2003) further investigate different remanufacturing strategies and the associated product and process characteristics. They conclude that return volume is still the major influential factor. The US Navy aviation depots are used as a typical MTO example in remanufacturing.

Another relevant work is the study by Souza and Ketzenberg (2002). They investigate an MTO remanufacturing system using a queueing network approach. Their focus is to determine the manufacturing–remanufacturing mixture in order to maximise the profit from strategic perspective. They assume that remanufacturing consumes less capacity than manufacturing, thus remanufacturing will still be attractive even if it has a less profit margin than manufacturing.

In addition to the above background, our current study is also motivated by the first author’s experiences with an engine remanufacturing company in the automotive industry. Due to high production costs, low production volume and high specification of the order, it is very difficult to forecast customer demand in terms of quantity and timing, and it would be rather hard to adapt a ‘‘base stock’’ model. Thus an MTO strategy has recently been adapted by the company so that a disassembly order is always released after a customer demand occurs. The aim of this paper is to build a simple model to estimate basic production planning and control parameters, such as the planned lead time in such an MTO remanufacturing environment. Results of this study are intended to be implemented in this engine remanufacturing company.

As will be seen later, our single component model turns out to have a structure similar to the well known ‘newsboy model’. Even though there is an extensive literature in this area, we have not been aware of any research concerning disassembly–remanufacturing systems using the newsboy model. For a comprehensive literature review concerning the newsboy problem, we refer to the study by Khouja (1999).

This paper is structured as follows. In Section 2 we provide a detailed description of the problem and related ordering policies in an MTO remanufacturing system with one single critical component to be recovered. In Section 3, we present our model formulation, including the optimisation condition for determining the control variables. The model is then extended to a two-component case in order to investigate how the production parameters influence decisions in Section 4. To support our analytical findings, numerical examples have been developed in Section 5. Finally, we draw some conclusions and provide suggestions for future studies.

2. Description of the problem

When we remanufacture high value items such as engines, the remanufacturing process is rather customer-order oriented. When an engine core is disassembled, it is essential to ensure that it is the right model to be supplied in the reassembly process. In addition, labour is often the major cost in acquiring the disassembled component. Therefore using a push strategy, i.e. disassembling components such as crankshaft and pushing them into the component stock, waiting for demand to occur, is usually very expensive. In this circumstance, an MTO strategy is often implemented instead. However, due to the uncertainty of the disassembly process, both in terms of its timing and component quality, it is very hard to determine the availability of a specific component. In case of shortage of the component from disassembly, a new component must be purchased to replace it. The question then becomes: At which point in time should we start the disassembly?

In particular, we refer to a disassembly–remanufacturing system with an MTO policy with material (of returned products and components) and information flow as illustrated in Fig. 1. The ordering procedure occurs according to the following consecutive steps:

i. At time point 0, a customer order is received and it should be (re)assembled at time T.
ii. A disassembly order is released at τ, with $0<\tau<T$, where τ is the decision variable, or alternatively, we consider the planned lead time $(T-\tau)$ as the decision variable.
iii. At $\tau+t_1$, the disassembly process is complete. The disassembly lead time t_1 is stochastic. Its density function and distribution function are $f(t)$ and $F(t)$, respectively, with $t\geqslant 0$. $f(t)$ is assumed to be piecewise continuous.
iv. At $\tau+t_1$, the status of the disassembled component is realised. It is either used for reassembly with probability p or disposed of due to its poor quality. p is the yield of return products.
v. If the quality of a component matches the standard, this component is held in inventory till time T for reassembly.
vi. If the disassembled component is disposed of, a purchasing order is released to procure a new component. This lead time t_2 is also stochastic. Its density and distribution functions are $g(t)$ and $G(t)$, respectively, with $t\geqslant 0$. $g(t)$ is also assumed to be piecewise continuous.
vii. If $\tau+t_1+t_2<T$, the component is held in the inventory till time T for reassembly.
viii. There is an inventory holding cost h (€/time unit) for the component during the interval $[\tau, T]$.

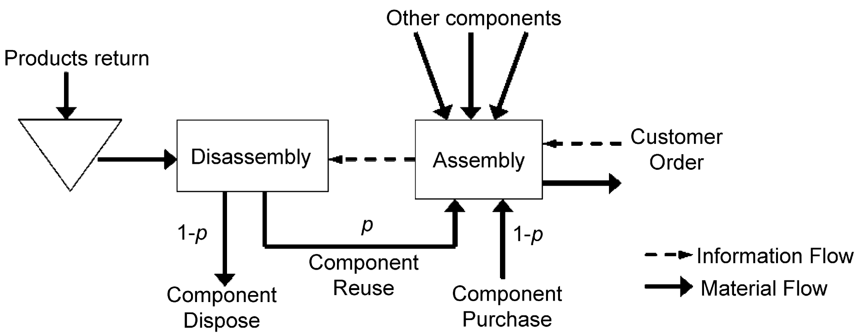

Fig. 1. Information and material flow in a disassembly–reassembly system.

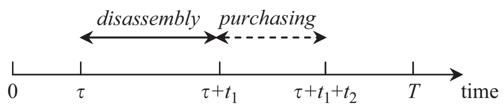

Fig. 2. Time scale of the ordering procedure.

ix. If the component is acquired after T, a penalty cost b (€/time unit) is incurred.

Fig. 2 illustrates the time scale of the above ordering procedure. The objective is then to minimise the sum of the total inventory holding cost between 0 and T, in addition to the penalty cost after T. Since the purchasing cost is proportional to $(1-p)$, it is a fixed cost in the long run. In this study, we therefore exclude the purchasing cost from our objective function.

3. Model formulation

The model can be formulated similarly to a newsboy problem. Even though the product to be disassembled is one unit, due to the yield probability and stochastic lead times of disassembly and purchasing, we either incur an overage cost if components are available before T, or an underage cost if components are not ready at time T.

3.1. Objective function

We first develop the objective function of the system described in the previous section. When one component is considered in the disassembly process, inventory only occurs during the interval $[\tau, T]$. At time t within this interval, we obtain inventory either if the disassembly process has finished and the quality is good, or when the disassembly process is finished, and the quality of the component is bad and a purchased component has been received.

The expected inventory is therefore

$$p\int_{t=\tau}^{T} F(t-\tau)\,\mathrm{d}t + (1-p) \times \int_{t=\tau}^{T}\int_{x=0}^{t-\tau} G(t-\tau-x)f(x)\,\mathrm{d}x\,\mathrm{d}t. \tag{1}$$

Similarly, the expected stockout is

$$p\int_{t=T}^{\infty}(1-F(t-\tau))\,\mathrm{d}t + (1-p) \times \int_{t=T}^{\infty}\left(1-\int_{x=0}^{t-\tau} G(t-\tau-x)f(x)\,\mathrm{d}x\right)\mathrm{d}t \tag{2}$$

which together with the inventory function in Eq. (1), provides the expected total cost for the particular order as

$$C = h\left(p\int_{t=\tau}^{T} F(t-\tau)\,\mathrm{d}t + (1-p) \times \int_{t=\tau}^{T}\int_{x=0}^{t-\tau} G(t-\tau-x)f(x)\,\mathrm{d}x\,\mathrm{d}t\right) + b\left(p\int_{t=T}^{\infty}(1-F(t-\tau))\,\mathrm{d}t + (1-p) \times \int_{t=T}^{\infty}\left(1-\int_{x=0}^{t-\tau} G(t-\tau-x)f(x)\,\mathrm{d}x\right)\mathrm{d}t\right). \tag{3}$$

We here exclude the purchasing cost, since on the long-run average this cost is fixed as we have mentioned in previous section.

3.2. Optimisation conditions

To simplify the analysis and develop the optimisation conditions, we introduce the following function.

Definition 1.

$$\Phi(t) = pF(t) + (1-p)\int_{x=0}^{t} G(t-x)f(x)\,\mathrm{d}x = pF(t) + (1-p)\int_{y=0}^{t}\int_{x=0}^{y} g(y-x)f(x)\,\mathrm{d}x\,\mathrm{d}y.$$

Lemma 1. *The function $\Phi(t)$ is a probability distribution function.*

Proof. According to Definition 1, it is easy to see that $\Phi(0)=0$ and $\Phi(\infty)=1$. In addition, the first-order derivative is $\mathrm{d}\Phi(t)/\mathrm{d}t = pf(t)+(1-p)\int_{x=0}^{t} g(t-x)f(x)\mathrm{d}x \geqslant 0$ making $\Phi(t)$ non-decreasing. □

Lemma 2. *$\phi(t) = \mathrm{d}\Phi(t)/\mathrm{d}t = pf(t) + (1-p)\int_{x=0}^{t} g(t-x)f(x)\,\mathrm{d}x$ is a probability density function.*

Inserting Definition 1, we can rewrite the objective function as

$$C = h\int_{t=\tau}^{T}\Phi(t-\tau)\,\mathrm{d}t + b\int_{t=T}^{\infty}(1-\Phi(t-\tau))\,\mathrm{d}t. \tag{4}$$

The first-order derivative of the objective function with respect to τ is

$$\frac{\mathrm{d}C}{\mathrm{d}\tau} = -h\int_{t=\tau}^{T} \phi(t-\tau)\,\mathrm{d}t + b\int_{t=T}^{\infty} \phi(t-\tau)\,\mathrm{d}t = -(h+b)\Phi(T-\tau) + b. \tag{5}$$

Theorem 1. *The objective function* (4) *is convex with respect to* τ.

Proof. The second-order derivative of C is $\partial^2 C/\partial\tau^2 = (h+b)\phi(T-\tau) \geqslant 0$. □

Theorem 2. *There exists an optimal solution* τ^* *that minimises the objective function. If* $-(h+b)\Phi(T)+b>0$, *then* $\tau^*=0$; *otherwise the necessary and sufficient optimisation condition is that the optimal* $\tau=\tau^*$ satisfies

$$\Phi(T-\tau^*) = \frac{b}{b+h}.$$

Proof. If $-(h+b)\Phi(T-\tau)+b>0$ starts by being positive at $\tau=0$, then this is the minimising value, otherwise, since $b/(h+b)<1$, there must exist an intersection between the graph of $\Phi(T-\tau)$ and $b/(h+b)$. This intersection may be an interval of τ. To the left of the intersection, C is decreasing and to the right C is increasing. □

From the above theorems and development, we notice that our model has a similar structure to the newsboy problem. The optimal planned lead time $(T-\tau)$ depends on the $b/(b+h)$ ratio and the distribution function $\Phi(t)$. In addition, we may have multiple solutions satisfying the above optimisation condition. In such a case, a short planned lead time is preferable from practical viewpoint.

3.3. Moments of the process lead time

In this section, we discuss the moments of the process lead time, which is defined as the time difference between releasing disassembly order and receiving the component, either from a disassembly or purchasing order. We should be aware that this lead time is a random variable and its value depends on the disassembly and purchasing lead times and the yield p. In principle (but with some exceptions, see Feller, 1966, pp. 222–224), if all the moments of this random lead time can be developed, the distribution function Φ(t) will be determined. Consequently, the optimal planned lead time can be calculated according to Theorem 2.

Theorem 3. *The first- and second-order moments of* $\phi(t)$ *can be written as*

$$\mu_\phi^1 = \mu_f + (1-p)\mu_g,$$
$$\mu_\phi^2 = \mu_f^2 + (1-p)(\mu_g^2 + 2\mu_f\mu_g).$$

Proof. The moments of $\phi(t)$ can be developed using the Laplace transform. Details (including high-order moments) are given in the Appendix A. □

Theorem 4. *The first- and second-order central moments of* $\phi(t)$ *can be written as*

$$\mu_\phi = \mu_f + (1-p)\mu_g,$$
$$\sigma_\phi^2 = \sigma_f^2 + (1-p)\sigma_g^2 + p(1-p)(\mu_g)^2.$$

The proof is again given in Appendix A.

3.4. Normal approximation

Due to the available moments and central moments from Theorems 3 and 4, it is possible to calculate the optimal planned lead time $T-\tau^*$ using approximations. In case an accurate planned lead time is needed, we could implement high-order moments and use a Pearson distribution (Grubbström and Tang, 2006). A different simple approach is using the normal distribution, which is often accepted in industrial practice as an approximation. A normal approximation is obtained using the first moment and second central moment $N(\mu_\phi, \sigma_\phi^2)$. Writing $\Omega(T-\tau|\mu_\phi, \sigma_\phi^2)$ for the distribution function, the solution will be approximated by

$$T-\tau = \Omega^{-1}\left(\frac{b}{h+b}\middle|\mu_\phi, \sigma_\phi^2\right) \tag{6}$$

which can be taken from standard normal tables. Alternatively, the optimal order release time can be written as a function of a safety factor k and the standard deviation of the lead time

$$T-\tau = \mu_\phi + k\sigma_\phi, \tag{7}$$

where the k value is acquired from a normal distribution table according to the ratio $b/(b+h)$. As an example of a normal approximation, if $h=1$,

$b=9$, $\mu_f=1$, $\mu_g=1$, $p=0.8$, $\sigma_f^2=1$, $\sigma_g^2=1$, then $\mu_\phi=1.2$, $\sigma_\phi^2=1.36$, and optimal $T-t=2.693$.

According to Theorem 4, the derivative of the mean value $\partial(\mu_\phi)/\partial p=-\mu_g<0$ whereas the derivative of the variance can be written as $\partial(\sigma_\phi^2)/\partial p=-\sigma_g^2+(\mu_g)^2-2p(\mu_g)^2$ with the sign undetermined. With a large purchasing lead time μ_g and a small σ_g^2, we could have $p<1/2-\sigma_g^2/2(\mu_g)^2$ and the above derivative is positive. At this level of p, increasing the disassembly yield would increase the variance of the lead times. Based on Eq. (7), we notice that with a high ratio $b/(b+h)$, we will obtain a large value of k and the second part $k\sigma_\phi$ will dominate. Thus an increasing p in this situation may lead to a longer planned lead time $(T-\tau)$, which is not intuitively straightforward.

4. Extension to two-component case

The idea here is to investigate situations when there are several important dissembled components to be used for further reassembly. For instance, what is the effect if one component has a high probability of failure? Shall we use the same order policy as in the previous section or make the purchase decision in advance and stock it as a normal inventory? The ordering procedure is similar to that suggested in the previous section, but we now assume that after the disassembly, two types of components are obtained (for instance crankshaft and engine box). The probabilities of having qualified components are p_1 and p_2, respectively. The density and distribution functions of the two corresponding lead times are now written $g_1(t)$, $G_1(t)$, $g_2(t)$, $G_2(t)$, respectively, the densities assumed piecewise continuous. A separate purchasing order will be released if a component is not qualified. Fig. 3 illustrates the time scale of the above ordering procedure, referred to the two-component case.

The inventory for component 1 may exist in the following cases: (i) before time point T, when disassembly has finished and the quality of component 1 is good, or the purchasing component has been received and (ii) after the time point T, when the component 1 has been received but component 2 is delayed. Its expected inventory is therefore

$$\begin{aligned}I_1 &= p_1\int_{t=\tau}^{T}F(t-\tau)\,\mathrm{d}t+(1-p_1)\\&\times\int_{t=\tau}^{T}\int_{x=0}^{t-\tau}G_1(t-\tau-x)f(x)\,\mathrm{d}x\,\mathrm{d}t\\&+p_1(1-p_2)\int_{t=T}^{\infty}\int_{x=0}^{t-\tau}(1-G_2(t-\tau-x))\\&\times f(x)\,\mathrm{d}x\,\mathrm{d}t+(1-p_1)(1-p_2)\\&\times\int_{t=T}^{\infty}\int_{x=0}^{t-\tau}G_1(t-\tau-x)\\&\times(1-G_2(t-\tau-x))f(x)\,\mathrm{d}x\,\mathrm{d}t.\end{aligned}\tag{8}$$

The expected inventory for component 2 has a similar structure with its index being switched. Meanwhile the expected stockout occurs only after time point T, if either the disassembly process is not finished or any of the components have not been received. Its expected value is written as

$$\begin{aligned}B &= \int_{t=T}^{\infty}(1-F(t-\tau))\,\mathrm{d}t+p_1(1-p_2)\\&\times\int_{t=T}^{\infty}\left(\int_{x=0}^{t-\tau}(1-G_2(t-\tau-x))f(x)\,\mathrm{d}x\right)\mathrm{d}t\\&+(1-p_1)p_2\int_{t=T}^{\infty}\left(\int_{x=0}^{t-\tau}(1-G_1(t-\tau-x))\right.\\&\left.\times f(x)\,\mathrm{d}x\right)\mathrm{d}t+(1-p_1)(1-p_2)\\&\times\int_{t=T}^{\infty}\left(1-\int_{x=0}^{t-\tau}G_1(t-\tau-x)\right.\\&\left.\times G_2(t-\tau-x)f(x)\,\mathrm{d}x\right)\mathrm{d}t.\end{aligned}\tag{9}$$

The total cost is the sum of inventory and stockout costs

$$C=h_1I_1+h_2I_2+bB.\tag{10}$$

Lemma 3. *If $G_1(t)$ and $G_2(t)$ are distribution functions, then $G_1(t)G_2(t)$ is a distribution function.*

Proof. Since $G_1(t)$ and $G_2(t)$ are both non-negative, non-decreasing functions, starting at zero and ending at unity, their product will behave the same way. Thus $G_1(t)G_2(t)$ is a distribution function. □

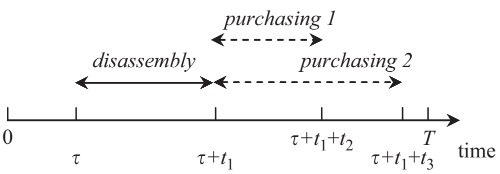

Fig. 3. Time scale of the ordering procedure in the two-component case: t_2 = purchasing lead time of component 1, t_3 = purchasing lead time of component 2.

Definition 2.

$$\Theta(t) = p_1 p_2 F(t) + (1-p_1)p_2 \int_{x=0}^{t} G_1(t-x)f(x)\,\mathrm{d}x + p_1(1-p_2)\int_{x=0}^{t} G_2(t-x)f(x)\,\mathrm{d}x + (1-p_1) \times (1-p_2)\int_{x=0}^{t} G_1(t-x)G_2(t-x)f(x)\,\mathrm{d}x.$$

Lemma 4. *$\Theta(t)$ is a distribution function.*

Proof. $\Theta(t)$ is a convex combination of four distribution functions, making it a distribution function. □

We then derive the first-order derivative of the objective function as

$$\frac{\mathrm{d}C}{\mathrm{d}\tau} = b - (h_1 + h_2 + b)\Theta(T-\tau). \tag{11}$$

Theorem 5. *The objective function* (10) *is convex and there exists an optimal solution τ^*. If $b-(h_1+h_2+b)\Theta(T)>0$, then $\tau^*=0$; otherwise the necessary and sufficient optimisation condition is that the optimal τ^* must satisfy*

$$\Theta(T-\tau^*) = \frac{b}{b+h_1+h_2}.$$

The proof follows the same approach as of Theorem 2.

Unfortunately, the Laplace transform of the last term (the "product term") in the function $\Theta(t)$ is more complicated than in the corresponding one-component case. Therefore, the moments of the process lead time are not easily obtained other than for simple distributions. For instance, for two uniform or two Gamma distributions, the moments of the product term may be derived in closed form (cf. Grubbström, 2005). However, the derivations and subsequent expressions become complicated. General expressions need still be developed (see also the Appendix A).

Despite the difficulties in finding general expressions for the moments of $\Theta(t)$, an optimal value of τ can always be obtained numerically when p_1, p_2, $F(t)$, $G_1(t)$ and $G_2(t)$ are given. Examples will be presented in Section 5.

5. Numerical examples

Numerical examples are used to illustrate our model and to investigate the behaviour of the disassembly–remanufacturing system proposed. First, we give some examples for the system with a single disassembled component and then we further address cases with two disassembled components.

We set the parameters for the base case as $b=10$, $h=1$, $T=10$ and $p=0.8$. The disassembly and ordering lead times are assumed to be normally distributed, with the following values: $\mu_f=4$, $\sigma_f=1$, $\mu_g=2$, and $\sigma_g=0.5$, respectively. According to Theorem 2 we have the optimal value $\tau^*=3.86$ for the single-component case.

An interesting investigation is the relative importance of p to the system performance and the decision variable. From a long run viewpoint, an increasing p reduces the cost for purchasing new components. However, it turns out that an increasing p does not necessarily decrease the inventory holding and stockout costs at the operational level (Fig. 4).

In many cases, an increasing p reduces the optimal planned lead time ($T-\tau$). However, as we have illustrated in Fig. 5, with a high ratio $b/(b+h)$, starting from a low p level, an increasing p slightly prolongs the planned lead time, see Fig. 5. This phenomenon consists with our discussion at the end of Section 3.4.

In the two-component cases, we have the following parameters for the base case: $b=10$, $h_1=1$, $h_2=2$, $\mu_f=4$, $\sigma_f=1$, and $\mu_{g1}=2$, $\sigma_{g1}=0.5$, $\mu_{g2}=4$, $\sigma_{g2}=0.5$, $T=10$, $p_1=0.8$ and $p_2=0.4$. Using Theorem 5 and substituting the optimal τ into

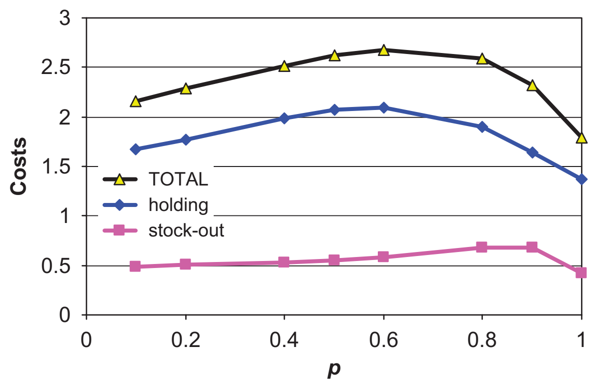

Fig. 4. Total costs and its components while varying probability p.

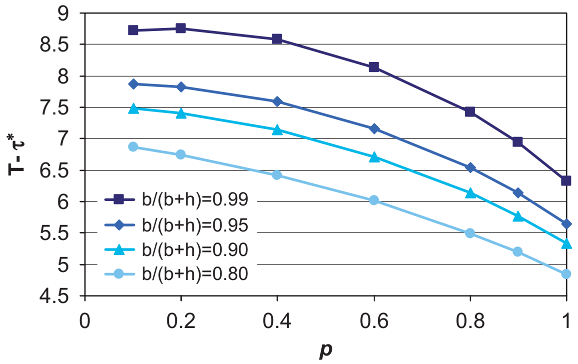

Fig. 5. The optimal planned lead time ($T-\tau$) while varying p at different levels of $b/(b+h)$ ratio with $h=1$ and b varying.

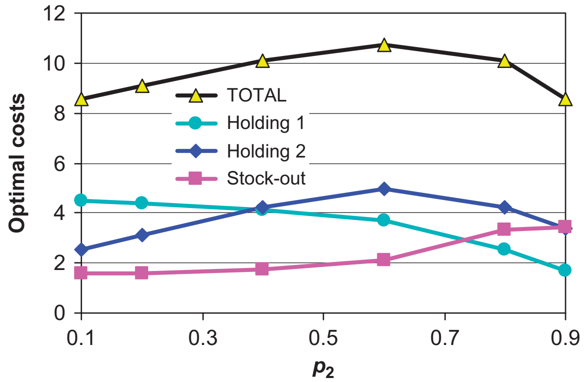

Fig. 6. Optimal total costs and its components while varying p_2.

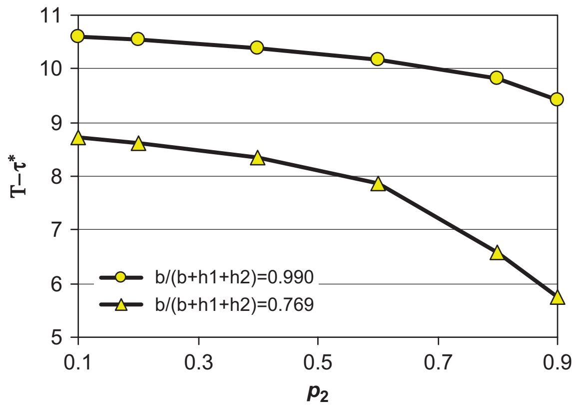

Fig. 7. Optimal instants of disassembly starting (τ^*) value while varying p_2.

the objective function, we obtain $\tau^*=1.659$ with a minimal cost $C^*=10.097$.

This two-component model can be used to determine the purchasing strategy. For instance, component 2 can be either purchased after or before disassembly. If it is purchased after disassembly, the optimal cost is $C^*=10.097$ as shown above. Instead, if component 2 is managed using a base stock policy and it is assumed always to be available in the purchased item inventory, we can apply our single-component model for component 1 and finally obtain the minimal cost to be $C'^*=2.939$. We may then determine the purchasing strategy by comparing the cost saving $\Delta C = C^* – C'^* = 7.158$ with the long-run average inventory holding cost of component 2. The latter cost is possible to estimate when this component has a fast moving pattern, i.e. it is used for several similar engine models.

Further illustrative examples for two-component cases are given in Figs. 6 and 7. The optimal total cost when varying p_2 has a similar pattern as in the single component case. However, the response of τ^* with respect of changing p_2 shows only an increasing function (Fig. 7). Further examples should be examined to investigate whether a similar property from Fig. 5 also exists in two-component cases or not.

6. Conclusions

In this paper, we have developed models for analysing a disassembly–remanufacturing system where production is driven by customer orders. We have investigated situations with both single and two critical components from disassembly. Both models turn out to be newsboy problems and the decision variable can be solved easily. In the single component model, we also present the moments and central moments, which can be further used as an approximation to obtain the planned lead time. This analysis also brings us some insights into the problem, for instance, an increasing p does not necessarily shorten the planned lead time. In addition, an increasing p does not necessarily reduce the system operational cost.

Our model can easily be implemented in practice, for instance, for setting the planned lead time in a production planning and control system and for determining the component purchasing strategy. The above issues are very important from the practical perspectives of disassembly and remanufacturing. The interesting findings in Figs. 4 and 5 are also important in practice. In an engine remanufacturing systems, the disassembly yield p is often associated with the labour skill. In the case company we have, the most skilled labour is assigned to the disassembly job in order to increase the yield. From the result of our model, we have to re-examine this policy. An appropriate range of p should be developed in order to allocate properly the production resources.

There are still many unsolved problems in similar systems. For instance, how does the component commonality influence the disassembly and remanufacturing decision? It is also very often the case that a system is a mixture of *MTO* and *MTS*. Then the question becomes: Where is the customer penetration point? If quantitative models are developed, we will gain a better understanding of the system and eventually improve the production planning and control of the disassembly–remanufacturing system.

Appendix A

A1. Proof of Theorem 3

Let $\tilde{\phi}(s)$ be Laplace transform of the corresponding probability density function, i.e.

$$\tilde{\phi}(s) = s\tilde{\Phi}(s) = \tilde{f}(s)(p + (1-p)\tilde{g}(s)).$$

Taking the nth derivative yields

$$\begin{aligned}\tilde{\phi}^{(n)}(s) &= \frac{\mathrm{d}\tilde{\phi}(s)}{\mathrm{d}s^n} = p\tilde{f}^{(n)}(s) + (1-p)\left(\tilde{f}(s)\tilde{g}(s)\right)^{(n)} \\ &= p\tilde{f}^{(n)}(s) + (1-p)\left(\sum_{i=0}^{n}\binom{n}{i}\tilde{f}^{(i)}(s)\tilde{g}^{(n-i)}(s)\right).\end{aligned}$$

Developing this expression, we have

$$\begin{aligned}\tilde{\phi}^{(0)}(s) &= p\tilde{f}(s) + (1-p)\tilde{f}(s)\tilde{g}(s), \\ \tilde{\phi}^{(1)}(s) &= p\tilde{f}^{(1)}(s) + (1-p) \\ &\quad \times\left(\tilde{f}^{(0)}(s)\tilde{g}^{(1)}(s) + \tilde{f}^{(1)}(s)\tilde{g}^{(0)}(s)\right), \\ \tilde{\phi}^{(2)}(s) &= p\tilde{f}^{(2)}(s) + (1-p)\left(\tilde{f}^{(0)}(s)\tilde{g}^{(2)}(s)\right. \\ &\quad \left.+2\tilde{f}^{(1)}(s)\tilde{g}^{(1)}(s) + \tilde{f}^{(2)}(s)\tilde{g}^{(0)}(s)\right), \\ \tilde{\phi}^{(3)}(s) &= p\tilde{f}^{(3)}(s) + (1-p)\left(\tilde{f}^{(0)}(s)\tilde{g}^{(3)}(s)\right. \\ &\quad \left.+3\tilde{f}^{(1)}(s)\tilde{g}^{(2)}(s) + 3\tilde{f}^{(2)}(s)\tilde{g}^{(1)}(s) + \tilde{f}^{(3)}(s)\tilde{g}^{(0)}(s)\right). \\ &\quad \ldots\end{aligned}$$

Since the limit $(-1)^n \lim_{s\to 0} \tilde{\phi}^{(n)}(s)$ yields the moments of the function $\phi(t)$, we have the moments of $\phi(t)$ written as

$$\begin{aligned}\mu_\phi^0 &= 1, \\ \mu_\phi^1 &= \mu_f + (1-p)\mu_g, \\ \mu_\phi^2 &= \mu_f^2 + (1-p)(\mu_g^2 + 2\mu_f\mu_g), \\ \mu_\phi^3 &= \mu_f^3 + (1-p)(\mu_g^2 + 3\mu_f\mu_g^2 + 3\mu_f^2\mu_g). \\ &\ldots \quad \square\end{aligned}$$

A2. Proof of Theorem 4

Using the definition and expression $\hat{\mu}_X^m = E[(X-\mu)^m] = \sum_{j=0}^{m}\binom{m}{j}(-\mu)^{m-j}\mu_X^j$ for the relation between moments and central moments gives us

$$\mu_\phi = \mu_f + (1-p)\mu_g,$$

$$\begin{aligned}\hat{\mu}_\phi^2 = \sigma_\phi^2 &= \mu_\phi^2 - (\mu_\phi)^2 \\ &= \mu_f^2 + (1-p)(\mu_g^2 + 2\mu_f\mu_g) - \left(\mu_f + (1-p)\mu_g\right)^2 \\ &= \sigma_f^2 + (1-p)(\sigma_g^2 + (\mu_g)^2) - (1-p)^2(\mu_g)^2 \\ &= \sigma_f^2 + (1-p)\sigma_g^2 + p(1-p)(\mu_g)^2\end{aligned}$$

and so on for higher order central moments. $\square$

A3. Derivation of moments in two-component case

$$\begin{aligned}\tilde{\phi}^{(n)}(s) &= \frac{\mathrm{d}\tilde{\phi}(s)}{\mathrm{d}s^n} = p\tilde{f}^{(n)}(s) + (1-p)\left(\tilde{f}(s)\tilde{g}(s)\right)^{(n)} \\ &= p\tilde{f}^{(n)}(s) + (1-p)\left(\sum_{i=0}^{n}\binom{n}{i}\tilde{f}^{(i)}(s)\tilde{g}^{(n-i)}(s)\right).\end{aligned}$$

The Laplace transform of the density of $\Theta(t)$ (Definition 2) is given by

$$\begin{aligned}\tilde{\theta}(s) = \pounds\left\{\frac{\mathrm{d}\Theta(t)}{\mathrm{d}t}\right\} &= p_1p_2\tilde{f}(s) + (1-p_1)p_2\tilde{g}_1(s)\tilde{f}(s) \\ &\quad + p_1(1-p_2)\tilde{g}_2(s)\tilde{f}(s) \\ &\quad + (1-p_1)(1-p_2)\tilde{h}(s)\tilde{f}(s),\end{aligned}$$

where $h(t) = \mathrm{d}H(t)/\mathrm{d}t = \mathrm{d}(G_1(t)G_2(t))/\mathrm{d}t$, $\tilde{h}(s) = \pounds\{\mathrm{d}h(t)/\mathrm{d}t\}$. The first three terms may be treated in the same way as in the proof of Theorem 3 above. The fourth term creates a need for additional developments (see Grubbström, 2005).

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