INT. J. PROD. RES., 2003, VOL. 41, NO. 5, 995–1016

Taylor & Francis
Taylor & Francis Group

Layout design in dynamic environments: strategies and quantitative indices

MARCELLO BRAGLIA†*, SIMONE ZANONI‡ and LUCIO ZAVANELLA‡

The design of a layout is a critical step, which may have a major influence on the success of a plant, particularly when demand is uncertain and variable. In order to counter this variability, one of two strategies is generally adopted, i.e. the identification of either a robust or an agile layout. This study proposes the adoption of indices that will help in identifying the strategy to be preferred. Starting from the case of a single row layout, the results of an experimental campaign are presented and discussed in order to show the efficacy of the indices themselves. Their calculation takes into account the profile of demand over time together with the product route sheets.

1. Introduction

The configuration of facilities is of considerable relevance for today's manufacturing systems, as it significantly affects production system performance. When facilities are optimally arranged, companies may reduce product costs, thus enhancing their competitive position. The Facility Layout Problem (FLP) is concerned with the identification of the most efficient arrangement of m resources, within a shop floor, according to suitable constraints (Meller and Gau 1996). The main FLP objective is to minimize the material handling cost between the machines, thus requiring, as a first step, the specification of the relative location of each resource. The second step is relevant to the solution of the detailed layout problem.

Nowadays, the success of a manufacturing organization is becoming increasingly linked to the design of a facility that is able to adapt quickly and effectively to technological changes and market requirements. In short, manufacturing facilities must be able to exhibit high levels of flexibility to react to significant changes in their operating requirements (Benjaafar and Sheikhzadeh 2000). The reasons described highlight the importance of, at the design stage, adopting a layout with a sufficient flexibility.

Flexible manufacturing systems (FMSs) have emerged in recent years as providing a strategic approach to production, capable of meeting the market requirement also for increased product variety, short product life cycles, and uncertain demand. From a strategic perspective, the definition of an efficient layout is a critical step in the implementation of an FMS, since the layout is difficult to design properly, and

Revision received May 2002.
† Dipartimento di Ingegneria Meccanica, Nucleare e della Produzione, Facoltà di Ingegneria, Università di Pisa, Via Bonanno Pisano, 25/B-56126 Pisa, Italy.
‡ Dipartimento di Ingegneria Meccanica, Facoltà di Ingegneria, Università degli Studi di Brescia, Via Branze, 38-25123 Brescia, Italy.
* To whom correspondence should be addressed. e-mail: m.braglia@ing.unipi.it

International Journal of Production Research ISSN 0020–7543 print/ISSN 1366–588X online © 2003 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00207540210162983

costly to modify (Afentakis *et al.* 1990), and it significantly affects the performance of the entire production system. It has been estimated that 20–50% of the total operating expenses within manufacturing operations are due to material handling, and it has been reported that effective layout design may reduce these costs by at least 10–30% (Tompkins and White 1996). In addition to material handling costs, the layout of a facility also impacts on production costs and work-in-process inventory levels (Rajasekharan *et al.* 1998).

The aim of the present study is to analyse the problem of defining the most efficient layout under uncertain market conditions, i.e. when demand significantly fluctuates over the planned time horizon (as also described in Hassan 1994). To this end, two alternative approaches are generally suggested (e.g. Kouvelis and Kiran 1991). The first one proposes the selection of a unique layout able to behave efficiently, even when mix and/or volume fluctuate (*robust* layout). The second approach (e.g. Kochhar and Heragu 1999) implies frequent layout modifications, carried out at each productive period affected by significant variations (*agile* layout). From an industrial point of view, the adoption of an agile layout also implies the availability of 'agile' resources, e.g. machine tools that can be easily relocated. This manufacturing philosophy has been preferred by some manufacturers proposing innovative machines, as has been particularly evident in recent years. It also suits the needs of assembly manufacturing units. On the other hand, the accurate definition of a robust layout becomes an obligatory approach when the manufacturing cycle involves resources that are not 'agile' at all (furnaces, presses etc).

According to this premise, the main target of the present study is to formulate indices and practices (as also discussed in Lin and Sharp 1999), that enable the forecasting and quantification of the benefits derivable from the adoption of a robust or an agile layout.

2. The reference situation

The method adopted for the investigation is mainly experimental and, in order to deal with an easy and understandable situation, the *single row machine layout problem* (SRMLP) will be discussed. The single row machine layout is a popular solution in manufacturing: the equipment is arranged along a straight line where a Material Handling Device (MHD) moves the items from one machine to another.

Even if the attention of the present study is paid to the SRMLP, a wider applicability to layout typologies may be foreseen, such as bidirectional and U-shaped loops. Furthermore, the single-row problem is generally considered as a convenient starting point for the development of researches on layouts, e.g. to propose and validate theories for modelling and performance evaluation. Nevertheless, particular layout configurations (e.g. a unidirectional loop, star layout) may require an objective function other than the one proposed in (1), even though the formulation of an optimal layout is still linked to the need for reducing the movement of the MHD between the machines (Meller and Gau 1996). In a deterministic environment, demand values are constant over the periods pertaining to the time horizon. Under the hypothesis of a fluctuating market demand, MHD movements are influenced by the predicted mix and the volumes for each period. The item flow, f_{ij}, between machine i and machine j may be more properly described by probability density functions (PDFs) defined as $\varphi_{ij}(x_{ij})$. Thus, x_{ij} is the stochastic variable describing the total flow, in both directions, between machine i and machine j

with $x_{ii} = 0$. These data may be grouped in the *flow matrix F*, whose dimension is $N \times N$, with N being the number of machines pertaining to the layout. Consequently, matrix F contains the expected values (i.e. the average) of the traffic between each pair of the N machines (Rosenblatt and Kropp 1992). In conclusion, the variability of the flows between machines is described by continuous probability distributions, thus originating the so-called *Stochastic Layout Problem* (SLP). An analytical formulation of the SLP may be found in Rosenblatt and Kropp (1992) and the flow matrix F sums up the total flows f_{ij} between machine i and machine j. Rosenblatt's approach aims to identify the most robust layout on the basis of the following objective function:

$$Z = \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{j=1}^{N} \sum_{m=1}^{N} A_{ikjm} Y_{ik} Y_{jm}, \tag{1}$$

where

$$Y_{ik} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to location } k \\ 0 & \text{otherwise} \end{cases}$$

$$A_{ikjm} = \begin{cases} x_{ij} d_{km} & \text{if } i \neq j \text{ or } k \neq m \\ x_{ii} d_{kk} + c_{ik} & \text{if } i = j \text{ and } k = m \end{cases}$$

c_{ik} are fixed costs due to the assignment of machine i to location k,
d_{km} is the distance between the machines assigned to location k and location m, with $d_{kk} = 0$.

For a given layout, the objective function becomes a function of the random variables, i.e. $Z = Z(x_{12}, x_{13}, \ldots, x_{1N}, x_{2N}, \ldots, x_{(N-1)N})$.

Once $P(F)$ is defined as the event probability associated with matrix F, it may be written that

$$P(F) = \prod_{\substack{i=1 \\ j=i+1}}^{N} \varphi_{ij}(x_{ij})\, \mathrm{d}x_{ij},$$

where $P(F)$ also represents the probability $P(Z) = P(F)$ that the objective function will yield a value equal to Z for matrix F. The aim of the Stochastic Layout Problem (SLP) is to minimize the total expected cost, i.e.

$$\operatorname{Min} \bar{Z} = \int \underset{S}{\ldots} \int Z(x_{12}, \ldots, x_{1N}, \ldots x_{(N-1)N}) P(Z)\, \mathrm{d}x_{12}\, \mathrm{d}x_{13} \ldots \mathrm{d}x_{1N} \ldots \mathrm{d}x_{(N-1)N} \tag{2}$$

Thanks to Rosenblatt's approach, it can be stated that it is possible to trace the solution of an SLP back to the deterministic situation. In particular, when the stochastic problem concerns the minimization of the expected cost of the material movement, the layout configuration minimizing it is the optimal layout for a quadratic assignment problem. This problem is based on the matrix of the averages of the distributions describing the flows between the layout resources.

Each $\varphi_{kr}(x_{kr})$ function is distributed according to a distribution with average (μ_{kr}) and variance (σ^2_{kr}) and the random variables x_{kr} are the sum of the random variables E_i describing the demand of the products over the T periods, i.e.

$$x_{kr} = \sum_{i=1}^{T} E_i X_{kr} \tag{3}$$

with

$$X_{kr} = \begin{cases} 1 & \forall k \text{ and } r \text{ successive in the } i\text{th product cycle,} \\ 0 & \text{otherwise.} \end{cases}$$

The conditions of a dynamic system may be effectively summed up by grouping parameters μ_{kr} and σ^2_{kr} in the two M and S^2 matrices:

$$M = \begin{bmatrix} 0 & \mu_{12} & \cdots & \mu_{1,N} \\ 0 & 0 & \cdots & \mu_{2,N} \\ \vdots & \vdots & \ddots & \mu_{(N-1),N} \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad \text{and} \quad S^2 = \begin{bmatrix} 0 & \sigma^2_{12} & \cdots & \sigma^2_{1,N} \\ 0 & 0 & \cdots & \sigma^2_{2,N} \\ \vdots & \vdots & \ddots & \sigma^2_{(N-1),N} \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

The two matrices represent the mean and the variance of the total material handled, between each pair of machines, in both directions of the single-row layout. The triangular aspect of the matrices is due to the fact that each term represents the total flow between two machines, irrespective of its direction (as in Rosenblatt and Kropp 1992). In conclusion, the single row configuration permits a triangular representation of the M and S^2 matrices.

In particular, if normal distributions are assumed, for each pair of machines k and r with $k \neq r$, the following relationship must hold:

$$\varphi_{kr}(x_{kr}) = \frac{1}{\sigma_{kr}\sqrt{2\pi}} \exp\left\{\frac{-(x_{kr} - \mu_{kr})^2}{2\sigma^2_{kr}}\right\}. \tag{4}$$

3. A simulation-based example

In order to illustrate the problem, we discuss a simple example, involving a five-machine plant manufacturing ten different types of products. Over the time horizon simulated, the demand for each product k varies according to a normal distribution of assigned average μ_k and variance σ^2_k. Each product has its own route sheet, i.e. a predetermined sequence of operations on the layout stations. During each period, material flows between stations are a consequence of the product demands and their route sheets. Simulation experiments were carried out by sampling the demand for each product, from its normal distribution, over 100 periods (e.g. weeks), thus obtaining the M and S matrices. Further data are represented by:

- the cost–area matrix, whose c_{ij} element indicates the cost of assigning machine i to location j, e.g. because of different space needs and/or availability of plant services;
- the dimension array, whose d_i element locates the position of the workpiece loading–unloading for machine i.

For the example proposed, the data commented upon above are presented here below. The data used have been chosen from a wide set of experiments, being representative and appropriate for comment.

N°PRODUCT	DEMAND μ	DEMAND σ	ROUTE-SHEET
1	415	11	5->3
2	61	11	1->4
3	351	13	4->1->5
4	7	0	5->1
5	562	193	1->2->5->4
6	762	121	5->3->1
7	617	89	3->2
8	299	99	4->1
9	900	275	1->4->5->3
10	41	6	3->5->2->3

COST AREA MATRIX

20	42	20	28	29
65	83	63	29	79
75	50	87	95	92
57	12	53	36	92
70	47	24	85	97

M - MATRIX

0	562	762	1611	358
0	0	658	0	603
0	0	0	0	2118
0	0	0	0	1462
0	0	0	0	0

S - MATRIX

0	193	121	293	13
0	0	89	0	193
0	0	0	0	301
0	0	0	0	336
0	0	0	0	0

DIMENSION ARRAY 2 5 1 1 1

Now, it is possible to evaluate the cost of each possible machine sequence, i.e. the layout. Let us compare the following layout sequences: 5-2-3-1-4 and 1-5-4-3-2. Normal distributions are used for demand to generate material flows for each simulated period. Then, the Total Transport Costs (TTC) are evaluated for each period, according to formula (1). The results are shown in figure 1.

It is evident that sequence 5-2-3-4-1 does not represent a successful solution, as the layout optimization requires a machine sequence that can provide an adequate performance for each period, even if demand is variable. This layout property, i.e. its '*robustness*', may be appreciated by computing the TTC average and variance for a sufficiently large number of simulations. For the case proposed, the most robust sequence found is 1-5-4-3-2, which gives an average cost equal to 247650. TTCs are lower than sequence 5-2-3-1-4 ones for each simulated period (figure 1). When comparing the performance of the entire set of 120 possible layouts, machine

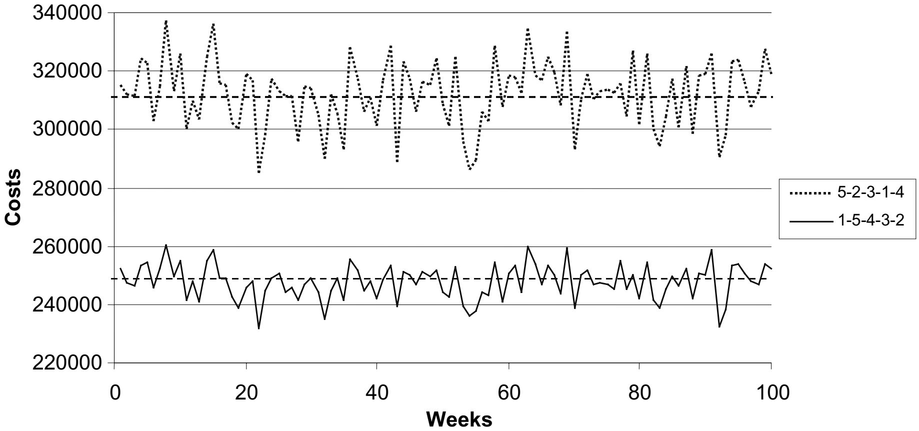


Figure 1. Total transport costs for machine sequences 5-2-3-1-4 and 1-5-4-3-2.

sequence 1-5-4-3-2 offered the best performance over 50 periods of the 100 simulated, while sequence 5-2-3-1-4 was never found as an optimal solution for any period (TTC = 312173).

The example shown highlights the impact of demand variability, from period to period, with respect to the adopted machine sequence. Of course, it is possible:

- to adopt a unique layout over the periods considered (robust approach) or, alternatively and whenever possible,
- to identify the best layout for each period and consequently relocate the machines (agile layout).

The strategy to be preferred should be identified a priori, on the basis of the expected working levels of the plant and the related economic consequences. Thus, there emerges the need for the formulation of an index that can describe and quantify the effectiveness of the plant reconfiguration.

4. Total penalty and re-layout

In order to appreciate the economic impact of demand variability on a given layout, let us define C_{ij} as the cost due to the utilization of layout i (i.e. optimized for the demand condition i) in the demand condition j. Thus, C_{jj} is the cost due to the utilization of layout j in the demand condition j and, obviously, $C_{ij} > C_{jj} \forall i \neq j$, as layout j is optimal for demand j. The Total Penalty TP_i associated with layout i may be defined as:

$$TP_i = \sum_{j=1}^{S} (C_{ij} - C_{jj}) = \sum_{j=1}^{S} C_{ij} - \sum_{j=1}^{S} C_{jj} \tag{5}$$

over the S possible scenarios of demand considered.

In other terms, the Total Penalty may be interpreted as the loss due to the adoption of the ith layout for all the productive scenarios when compared with the agile strategy. The Total Penalty calculated with respect to the robust layout becomes an important reference, as it may be defined as the *maximum re-layout costs* *acceptable to support the agile strategy*. Thanks to this concept, it is possible to determine whether a robust or an agile layout is more suitable for a predetermined plant, once given the related demand. It follows that:

- the more robust the layout, the lower the need for its re-layouts, i.e. an extremely robust layout does not require the system agility;
- a poor robustness of a layout may be interpreted as the need for an agile plant, suitable for frequent redefinitions of the machine sequences.

To this end, the following normalized index may be introduced:

$$0 \le NS_{\text{norm}} = \frac{\sum_{i=1}^{S} NS_i}{S} \frac{1}{N} \le 1, \tag{6}$$

where

NS_i	is the number of machines to be relocated in period i, so as to shift from the robust to the layout optimal for the same ith period;
S	is the number of periods simulated over the time horizon;
N	is the number of machines of the plant.

Situations where the robust layout is affected by a large NS_{norm} value will probably require agility and, therefore, not be suitable for the adoption of an optimization based on robustness (it is possible to state that the problem itself is *not robust*). On the contrary, situations described by low NS_{norm} values may be solved by the adoption of the most robust layout.

In such a context, it is essential to evaluate the mechanism leading to the plant re-layout. A series of simulation tests was carried out to perceive the link between the demand profile and the Re-layout Expectation Level (REL) of a given system, as quantified later by the introduction of the REL definition. For each single problem, the following parameters were adopted to describe the data structure concisely:

- the mean μ_μ of the averages of the distributions (as also in matrix M):

$$\mu_\mu = \frac{\sum_{i=1}^{N} \sum_{j=i+1}^{N} \mu_{ij}}{\frac{N(N-1)}{2}} \tag{7}$$

- the standard deviation σ_μ of the averages of the distributions, i.e. the standard deviation of matrix M:

$$\sigma_\mu = \sqrt{\frac{\sum_{i=1}^{N} \sum_{j=i+1}^{N} (\mu_{ij} - \mu_\mu)^2}{\frac{N(N-1)}{2}}}, \tag{8}$$

- the mean μ_σ of the standard deviations of the distributions, i.e. the average of the S matrix elements:

$$\mu_\sigma = \frac{\sum_{i=1}^{N}\sum_{j=i+1}^{N}\sigma_{ij}}{\frac{N(N-1)}{2}}, \tag{9}$$

- and the related standard deviation σ_σ of S matrix data:

$$\sigma_\sigma = \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=i+1}^{N}\left(\sigma_{ij}-\mu_\sigma\right)^2}{\frac{N(N-1)}{2}}}. \tag{10}$$

According to the problem outlined, an experimental campaign was organized as follows.

(1) Considering the SRMLP, 200 situations are generated for $N = 5$ machines.
(2) The M and S matrices are randomly generated according to different μ_μ values and identical σ_μ, μ_σ, σ_σ parameters.
(3) Each single layout problem is simulated over 100 periods and the REL of the layout is evaluated by parameter $NS_{\text{med}} = N \cdot NS_{\text{norm}}$.
(4) The experiment is repeated, from point (1), for the other σ_μ, $\mu - \sigma$, σ_σ parameters, i.e. one is sampled from a distribution and the three left are fixed.

The four analyses discussed hereafter are commonly based on the following data set.

```
Number of problems = 200
Simulation horizon = 100 periods
Number of machines = 5
Assignment costs = uniformly variable between 10 and 100
Distances = uniformly variable between 1 and 5
```

It should be noted that data above and following (see figure 2) have been chosen from the set of experiments carried out and all of them show the same trend. The results obtained are suitable for general comment, and they shape the whole set of experiments carried out. Below, four cases are discussed, as four parameters are still to be assigned. For each case, one of the parameters will be sampled from a distribution, while the remainder will be kept fixed. Of course, if one of the fixed parameters is influential, it will impact on the absolute position of the results obtained, but not on the shape of their distribution. The results of the study are shown in figure 2, which also gives the correlation coefficient for each relationship studied.

The experiments carried out showed how the parameters most influential in determining the REL of a layout are σ_μ and μ_σ:

- the former expresses the 'closeness' of data pertaining to matrix M: the closer the averages of the distributions, the higher the tendency of the machines to require relocation;
- the latter states that a layout problem may lack robustness because of the increase in the absolute variability of the flow distributions.

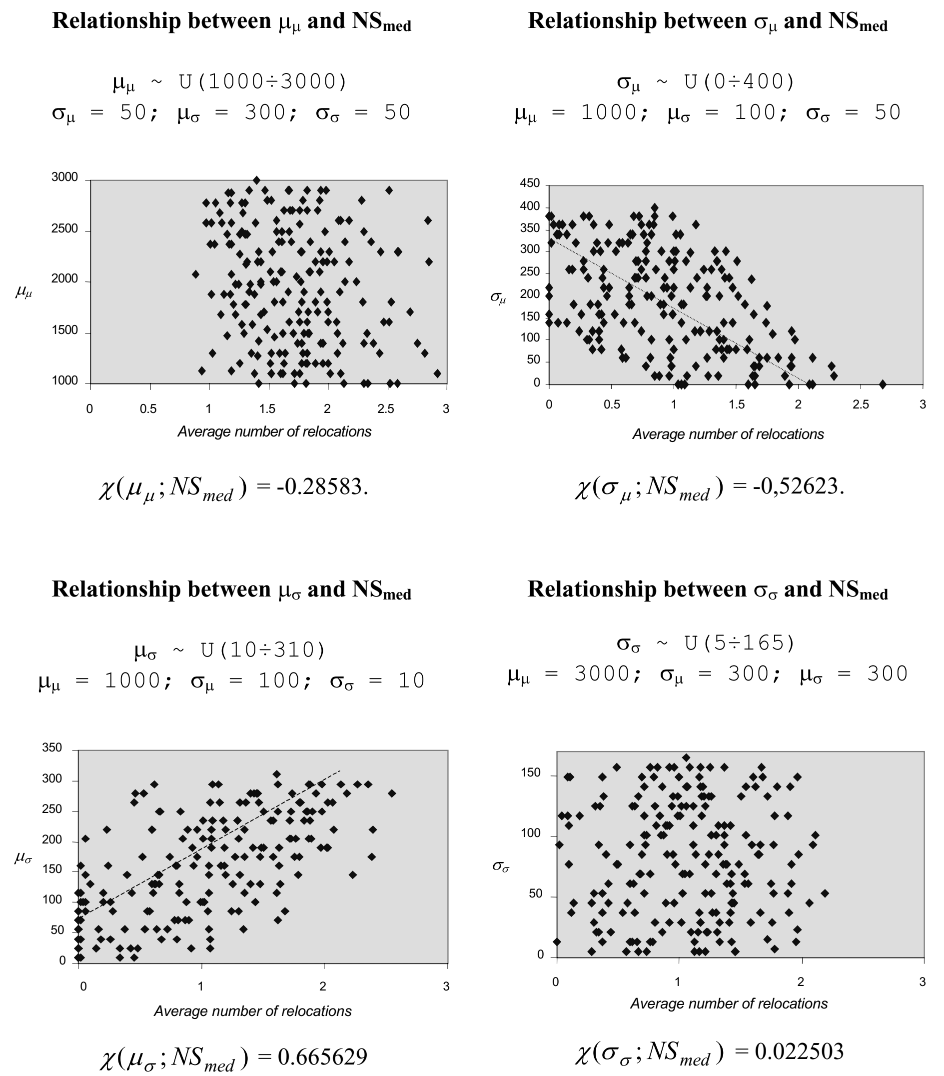


Figure 2. Relationships between μ_μ, σ_μ, μ_σ, σ_σ and the average number of relocations for (200 × 4) stochastic SRMLP.

The following sections will try to interpret these preliminary results, drawing conclusions useful in foreseeing the criticality of a layout problem, independently of the adoption of simulation tools.

5. Re-layout mechanisms

The results proposed in the previous section highlighted that, while decreasing σ_μ and increasing μ_σ, the system will require additional machine relocations.

This is due to the fact that each single flow distribution is inclined to overlap other flow distributions. In other words, relative alterations of the flows between machines are more likely to occur, over the time horizon, and the problem cannot be robust.

In order to appreciate this phenomenon, it should be remembered that variable x_{ij} represents the sum of the flows of various products that are presented consecutively, in their route-sheets, between machine i and machine j.

It is possible to calculate the probability of non-interference between the two generic flows x_{ij} and x_{kr} via the random variable y, defined as follows:

$$y \sim N[(\mu_{ij} - \mu_{kr});\ (\sigma_{ij}^2 + \sigma_{kr}^2)]. \tag{11}$$

However, it should be pointed out that the production schedule may require that one or more products, on account of their route-sheets, have to visit consecutively both machines i and j and machines k and r. Let us group these products in family $\tilde{P}$.

Of course, the contribution of products pertaining to family $\tilde{P}$ has to be carefully considered, while calculating the probability of non-interference between flows x_{ij} and x_{kr}. In fact, if family $\tilde{P}$ is not empty, the single components of the relative flows are not completely independent, as a variation of the level of production of $\tilde{P}$ products impacts on both of these flows. Before calculating the non-interference probability, this fact may be taken into account by neglecting the contribution to the relative flows of products pertaining to $\tilde{P}$.

Now, it is possible to define a random variable $\tau_{ij,kr}$, as the sum of the random variable x_{ij} due to the products pertaining to set $\tilde{P}$; this involves summing up all the flows, which are mutually dependent, shown in the matrix of table 1.

In particular, each random variable $\tau_{ij,kr}$ is characterized by a mean and a variance, given respectively by:

$$\mu_{\tau_{ij,kr}} = \sum_{\text{products } \tilde{\text{P}}} \mu_{\text{x}_{\text{ij}}}, \tag{12}$$

$$\sigma^2_{\tau_{ij,kr}} = \sum_{\text{products } \tilde{\text{P}}} \sigma^2_{\text{x}_{\text{ij}}}. \tag{13}$$

Once all the elements of the table 1 matrix are known, it is possible to calculate the corrected probability of non-interference between the two generic flows x_{ij} and x_{kr} via the corrected random variable y_c, defined as follows:

$$y_c \sim N[(\mu_{ij} - \mu_{kr});\ (\sigma_{ij}^2 + \sigma_{kr}^2 - 2\sigma^2_{\tau_{ij,kr}})]. \tag{14}$$

It should be noted that the mean of y_c is identical to the mean of y. In fact, the contributions of the two mutually dependent flows are present, in both of the μ_{ij} and

	x_{12}	x_{13}	x_{14}	$\cdots$	x_{1N}	x_{23}	x_{24}	$\cdots$	$x_{(N-1)N}$
x_{12}	–	$\tau_{12,13}$	$\tau_{12,14}$	$\cdots$	$\tau_{12,1N}$	$\tau_{12,23}$	$\tau_{12,24}$	$\cdots$	$\tau_{12,(N-1)N}$
x_{13}	$\tau_{13,12}$	–	$\tau_{13,14}$	$\cdots$	$\tau_{13,1N}$	$\tau_{13,23}$	$\tau_{13,24}$	$\cdots$	$\tau_{13,(N-1)N}$
x_{14}	$\tau_{14,12}$	$\tau_{14,13}$	–	$\cdots$	$\tau_{14,1N}$	$\tau_{14,23}$	$\tau_{14,24}$	$\cdots$	$\tau_{14,(N-1)N}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	–	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{1N}	$\tau_{1N,12}$	$\tau_{1N,13}$	$\tau_{1N,14}$	$\cdots$	–	$\tau_{1N,23}$	$\tau_{1N,24}$	$\cdots$	$\tau_{1N,(N-1)N}$
x_{23}	$\tau_{23,12}$	$\tau_{23,13}$	$\tau_{23,14}$	$\cdots$	$\tau_{23,1N}$	–	$\tau_{23,24}$	$\cdots$	$\tau_{23,(N-1)N}$
x_{24}	$\tau_{24,12}$	$\tau_{24,13}$	$\tau_{24,14}$	$\cdots$	$\tau_{24,1N}$	$\tau_{24,23}$	–	$\cdots$	$\tau_{24,(N-1)N}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	–	$\cdots$
$x_{(N-1)N}$	$\tau_{(N-1)N,12}$	$\tau_{(N-1)N,13}$	$\tau_{(N-1)N,14}$	$\cdots$	$\tau_{(N-1)N,1N}$	$\tau_{(N-1)N,23}$	$\tau_{(N-1)N,24}$	$\cdots$	–

Table 1. Matrix of mutually dependent flows.

μ_{kr} terms, with opposite signs. Therefore, the mutual dependencies are implicitly suppressed. As far as the variance is concerned, the contribution related to mutually dependent flows appears, with the same sign, in both of the σ_{ij} and σ_{kr} terms. Consequently, this contribution has been subtracted twice in the formula of y_c variance.

Now, it is possible to estimate effectively, i.e. without interferences, the probability that two flow distributions do not overlap in a problem of a stochastic layout. Furthermore, it is possible to quantify the robustness of a stochastic layout problem by avoiding a simulation-based approach. The superimposition level $\phi_{ij,kr}$ for each distribution couple x_{ij} and x_{kr} pertaining to the flow matrix can be calculated as follows:

$$\phi_{ij,kr} = P(y_c \geq 0) = \int_0^{+\infty} \frac{1}{\sqrt{2\pi(\sigma_{ij}^2 + \sigma_{kr}^2 - 2\sigma_{\tau_{ij,kr}}^2)}} \exp\left(- \frac{[y - (\mu_{ij} - \mu_{kr})]^2}{2(\sigma_{ij}^2 + \sigma_{kr}^2 - 2\sigma_{\tau_{ij,kr}}^2)} \right) \mathrm{d}y. \tag{15}$$

Obviously, the lower the superposition level, the greater the robustness of the problem.

Given a layout problem described by an $F_{\text{stochastic}}$ matrix, it is possible to arrange a further matrix that has exactly the same format as the matrix of table 1, with elements $\phi_{ij,kr}$ instead of $\tau_{ij,kr}$.

The properties of these elements are:

- $\phi_{ij,kr} \in [0, 1] \quad \forall i, j, k, r,$
- $\phi_{ij,kr} + \phi_{kr,ij} = 1 \qquad \text{if } x_{ij} \neq x_{kr} \quad \forall i, j, k, r.$

The first property is a plain consequence of the definition of $\phi_{ij,kr}$, while the second one depends on the fact that:

$$\phi_{ij,kr} + \phi_{kr,ij} = P(x_{ij} > x_{kr}) + P(x_{kr} < x_{ij}) = P(x_{ij} > x_{kr}) + P(x_{ij} < x_{kr}) = 1.$$

In other words, the symmetric elements of table 2 are complementary ones, thus making only half of the table elements relevant to the end of the index calculation. For each pair of values, let us introduce the following substitution:

$$\psi_{ij,kr} = \psi_{kr,ij} = MAX[\phi_{ij,kr}; \phi_{kr,ij}] \text{ so that } 0.5 \leq \psi_{ij,kr} \leq 1. \tag{16}$$

It should be noted that, in the limiting case of two deterministic flows, $\phi_{ij,kr} = \phi_{kr,ij} = 1$ and according to (16), $\psi_{ij,kr} = \psi_{kr,ij} = 1$.

	x_{12}	x_{13}	x_{14}	…	x_{1N}	x_{23}	x_{24}	…	$x_{(N-1)N}$
x_{12}	–	$\phi_{12,13}$	$\phi_{12,14}$	…	$\phi_{12,1N}$	$\phi_{12,23}$	$\phi_{12,24}$	…	$\phi_{12,(N-1)N}$
x_{13}	$\phi_{13,12}$	–	$\phi_{13,14}$	…	$\phi_{13,1N}$	$\phi_{13,23}$	$\phi_{13,24}$	…	$\phi_{13,(N-1)N}$
x_{14}	$\phi_{14,12}$	$\phi_{14,13}$	–	…	$\phi_{14,1N}$	$\phi_{14,23}$	$\phi_{14,24}$	…	$\phi_{14,(N-1)N}$
…	…	…	…	–	…	…	…	…	…
x_{1N}	$\phi_{1N,12}$	$\phi_{1N,13}$	$\phi_{1N,14}$	…	–	$\phi_{1N,23}$	$\phi_{1N,24}$	…	$\phi_{1N,(N-1)N}$
x_{23}	$\phi_{23,12}$	$\phi_{23,13}$	$\phi_{23,14}$	…	$\phi_{23,1N}$	–	$\phi_{23,24}$	…	$\phi_{23,(N-1)N}$
x_{24}	$\phi_{24,12}$	$\phi_{24,13}$	$\phi_{24,14}$	…	$\phi_{24,1N}$	$\phi_{24,23}$	–	…	$\phi_{24,(N-1)N}$
…	…	…	…	…	…	…	…	–	…
$x_{(N-1)N}$	$\phi_{(N-1)N,12}$	$\phi_{(N-1)N,13}$	$\phi_{(N-1)N,14}$	…	$\phi_{(N-1)N,1N}$	$\phi_{(N-1)N,23}$	$\phi_{(N-1)N,24}$	…	–

Table 2. Superimposition of the flow distributions for $F_{\text{stochastic}}$ matrix.

This simplification allows the evaluation of an index of the problem robustness by the mean of $\psi_{ij,kr}$ values, i.e. for an N-machine problem:

$$Layout\ Problem\ Robustness\ Index\ LPRI = \frac{\sum_{i=1}^{N}\sum_{j=i+1}^{N}\sum_{\substack{k=1\\k\neq i}}^{N}\sum_{\substack{r=k+1\\r\neq j}}^{N}\psi_{ij,kr}}{N_{TOT}}. \qquad (17)$$

with

$$N_{TOT} = \left(\frac{N(N-1)}{2}\right)^2 - N$$

and, obviously, $0.5 \leq LPRI \leq 1$.

The larger the LPRI value, the lower the superimposition between the distributions and, consequently, the higher the problem robustness. The previous statement was verified by the following experiment:

(1) LPRI calculation by numerical integration;
(2) simulation of the plant working conditions over several periods;
(3) calculation of the normalized average number of machine relocations NS_{norm};
(4) comparison between LPRI and NS_{norm}.

For the sake of brevity, results offered hereafter refer only to the five-machine case.

```
Number of problems = 1000
Simulation horizon = 100 periods
Number of products = 20
Length of Route-sheet = 2 ÷ 5
Max demand for each product = uniformly distributed between 0 and 1000
Assignment costs = uniformly distributed between 100 and 1000
Distances = uniformly variable between 1 and 5
```

The diagram shown in figure 3 represents all the 1000 values of LPRI with respect to the related NS_{norm} value.

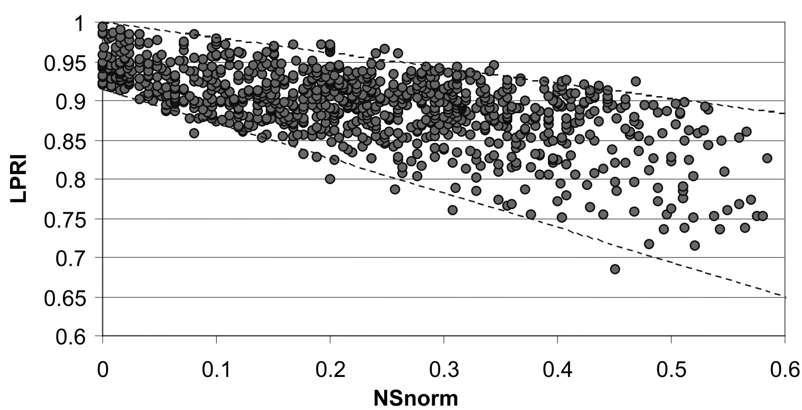
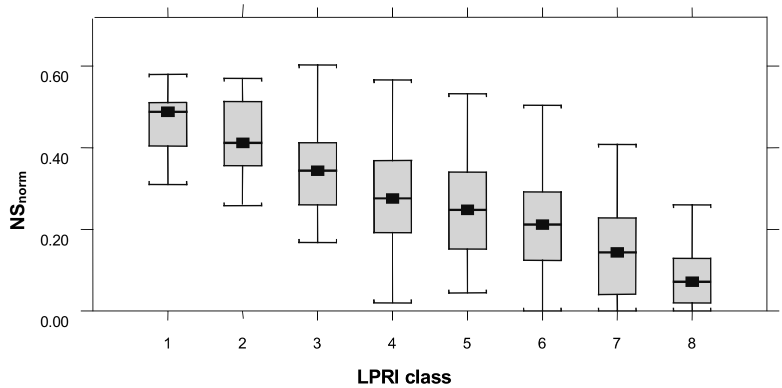


Figure 3. LPRI values versus NS_{norm} for the five machine problem.



LPRI Class	LPRI Range	$\overline{\text{LPRI}}$
1	< 0.8	0.765
2	0.8 - 0.825	0.804
3	0.825 - 0.85	0.840
4	0.85 - 0.875	0.868
5	0.875 - 0.9	0.892
6	0.9 - 0.925	0.913
7	0.925 - 095	0.930
8	> 0.95	0.967

Figure 4. Box-plot of LPRI values versus NS_{norm} for the five machine problem.

Data may be arranged in classes of LPRI width equal to 0.025, as in figure 4; for each class, the value of the average NS_{norm} and the respective value of the mean LPRI ($\overline{\text{LPRI}}$) were calculated. The results of the previous experimental campaign were rearranged according to the scheme suggested and grouped into classes, thus obtaining the box plot of figure 4 (more details may be found in Law and Kelton 1994).

The results of figure 4 are characterized by a significant correlation between the values of the two parameters examined. In fact, the correlation coefficient between the average NS_{norm} and the LPRI value of the correspondent class is equal to:

$$\chi_{(\text{LPRI; NSnorm})} = 0.996.$$

The simulations presented lead us to draw the conclusion that the LPRI value clearly identifies the system attitude to be redefined.

6. Measuring the robustness of a layout

In a layout problem, each single solution must be evaluated from an economic point of view. This consideration acquires particular relevance when a dynamic context is considered. A typical approach is represented by the calculation of the Total Penalty Cost (TPC) (Rosenblatt and Lee 1987), which represents, in its simplest formulation, the cost due to transportation when layout, route sheets and mix dynamics over time are assigned. Of course, the TPCs may be compared with the ones associated with the agile solution. The result of the comparison estimate the expected inefficiency of the robust layout with respect to the lowest level of costs. The basic limitation of this approach is that:

- it always requires a simulation-based analysis and it is also necessary to compare agile and robust solutions over the given time horizon;
- thus, it is computationally burdensome;
- it is not normalized, but compares the performance of one layout to another, without providing any absolute information.

To this end, let us modify the objective function proposed in section 2. The following substitution is introduced:

$$Y_{ik} Y_{jm} \Rightarrow d_{ij}(l),$$

where $d_{ij}(l)$ is the distance between machine i and machine j in the lth layout. These distances will represent the new variables with respect to which the system optimization will be pursued. The optimization will require the minimization of the expected costs, without considering assignment costs. In this case, the number of machines N is given, together with their relevant dimension (D_i for machine i).

Thus, the following linear formulation of the stochastic SRMLP is introduced:

$$\text{Min}\,\overline{Z} = \int \underset{S}{\cdots} \int Z(x_{12}, \ldots, x_{1N}, x_{(N-1)N}) P(Z)\, \mathrm{d}x_{12} \ldots \mathrm{d}x_{1N} \ldots \mathrm{d}x_{(N-1)N}, \tag{18}$$

where:

$$Z = \sum_{i=1}^{N} \sum_{j=i+1}^{N} x_{ij} d_{ij}(l), \tag{19}$$

$$d_{ij}(l) \geq \frac{D_i + D_j}{2}, \tag{20}$$

$$d_{ij}(l) \leq \sum_{k=1}^{N} D_k - \left(\frac{D_i + D_j}{2}\right), \tag{21}$$

$$\sum_{i=1}^{N} \sum_{j=i+1}^{N} Y_{ij} = N - 1 \text{ with } Y_{ij} = \begin{cases} 1 & \text{if } d_{ij}(l) = \dfrac{D_i + D_j}{2}, \\ 0 & \text{otherwise,} \end{cases} \tag{22}$$

$$P(Z) = \prod_{\substack{i=1 \\ j=i+1}}^{N} \varphi_{ij}(x_{ij})\, \mathrm{d}x_{ij}, \tag{23}$$

$$x_{ij} : \text{integer.} \tag{24}$$

Under the assumption of flows distributed according to normal distributions, it may be experimentally shown that the cost function of the layout problem is also distributed normally. To this end, let us consider the situation of a seven-machine stochastic layout, dealing with 20 product types. Each product has a route-sheet, which varies from two to five visited machines. The product demand is assumed to be distributed normally, the average of each distribution is sampled from a uniform distribution, with limits 0 and 1000. The related standard deviation is also sampled from a uniform distribution with limits 0 and 0.7 times the average previously set.

For the example considered, the M and S matrices are shown below, together with the machine dimensions:

M - MATRIX

0	883	705	0	1719	2579	352
0	0	911	1150	2064	970	325
0	0	0	944	493	191	0
0	0	0	0	1716	1980	2278
0	0	0	0	0	1841	1249
0	0	0	0	0	0	3505
0	0	0	0	0	0	0

S - MATRIX

0	1	151	0	64	201	37
0	0	72	59	161	158	41
0	0	0	130	49	25	0
0	0	0	0	191	119	287
0	0	0	0	0	22	73
0	0	0	0	0	0	325
0	0	0	0	0	0	0

DIMENSION ARRAY 1 5 9 7 2 10 4

For the case proposed, the most robust sequence found is 6-7-1-5-4-2-3. Given this sequence, it is possible to calculate:

$$M(l) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \mu_{ij} d_{ij}(l) = 487643 \quad \text{and} \quad S(l) = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \sigma_{ij}^2 d_{ij}^2(l) = 229159044.$$

The cost distribution for a simulation over 2000 periods is offered in figure 5.

The average of the simulation results is equal to 492 193, i.e. an error of 0.93% with respect to $M(l)$, and the standard deviation is 15 411, with an error of 1.8% with respect to $\sqrt{S(l)}$.

The normal distribution of costs suggests the utilization of parameter $M(l)$ as a relative measurement of the robustness of the solutions. In fact, each layout configuration presents a normally distributed cost function with average $M(l)$. Thus, the most robust layout, i.e. the one with the lowest expected cost, will be the layout associated with the $d_{ij}(l)$ values that minimize equation (19). It will be the layout calculated with respect to the average of the flow distributions of the stochastic problem. However, the robustness of a layout is an absolute concept: it is the ability of an optimal layout to be competitive with respect to the agile strategy. This comparison must be carried out by computer-based simulations. Hence, an absolute index of robustness, which does not require the simulation approach, is proposed in the following section.

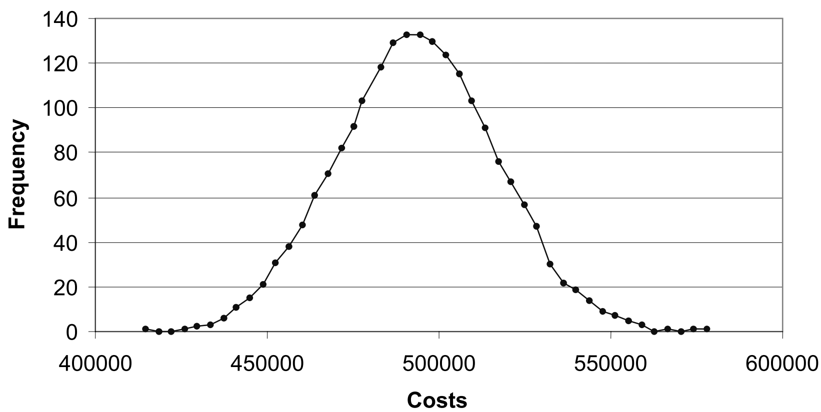


Figure 5. Cost distribution for 6-7-1-5-4-2-3 sequence over 2000 periods.

7. The Robustness Layout Index (LCRI)

For a given layout, it is possible to evaluate the probability that its cost function is larger than those of other layout configurations, so as to evaluate the probability that the given layout may prove inefficient with respect to the other layouts. The results obtained are a statistical measure of the robustness of a configuration.

Given a stochastic SRMLP with N machines, where the costs of assigning machines to areas are not considered, the admissible solution space W has dimension $N!/2$, since symmetric solutions coincide. The following notation may be introduced for layout $l \in W$:

- $C(l)$ random variable describing the cost function of the lth layout (mean $M(l)$, variance $S(l)$);
- $\varphi(l)$ pdf of the $C(l)$ cost function.

When, for $m \in W$ and $m \neq l$, the probability

$$P(C(l) \geq C(m)) \tag{25}$$

is calculated, it is the same as calculating the probability that the mth layout is optimal with respect to the lth one. Of course, a configuration offering a low probability of inefficiency, with respect to all the others, will be a highly robust one. In order to evaluate equation (25)'s probability, the following function is defined:

$$y = C(l) - C(m), \tag{26}$$

thus obtaining

$$P(C(l) \geq C(m)) = P(y \geq 0). \tag{27}$$

Under the hypothesis that both $C(l)$ and $C(m)$ are normally distributed, the random variable y will also be a normal one, i.e.:

$$y \equiv N[(M(l) - M(m)); (S(l) + S(m))],$$

and formula (27) may be written as

$$P(y \geq 0) = \vartheta_{l,m} = \int_0^{+\infty} \frac{1}{\sqrt{2\pi[S(l) + S(m)]}} \exp\left(-\frac{\{y - [M(l) - M(m)]\}^2}{2[S(l) + S(m)]} \right) \mathrm{d}y. \tag{28}$$

According to the definition of $\vartheta_{l,m}$, it must hold that:

$$0 \leq \vartheta_{l,m} \leq 1. \tag{29}$$

Therefore, it is possible to estimate the probability that the given solution maintains a lower cost of transport, in any production scenario compatible with the distributions of the product demands and with respect to any other generic configuration. Nevertheless, such a probability refers only to the relative comparison of two configurations: the next step is to formulate an absolute index of robustness for each single configuration of a stochastic layout.

As shown experimentally in Malmborg and Bukhary (1997), Malmborg *et al.* (1998) and Malmborg (1999), a single layout sequence presents a cost distribution of normal type. In addition, the costs of all the possible configurations will also present a normal distribution (in particular, when the distances between any two adjacent machines are the same). According to this observation, a random variable may be introduced to represent the cost of all the possible configurations of a layout. Such a random variable will be distributed with a mean and variance given by:

$$\overline{M(l)} = \frac{\sum_{l=1}^{N!} M(l)}{N!}, \quad \overline{S(l)} = \frac{\sum_{l=1}^{N!} S(l)}{N!}.$$

Moreover, given an N machine layout problem, in which the costs of allocation of the machine to the different area are not considered, the space of the possible solutions will again have a dimension of $N!/2$, since symmetrical solutions coincide. Hence, it will be possible to halve the exhaustive calculation of the variable that represents the cost.

In conclusion, it is possible to calculate:

- the probability that a generic layout will accrue costs lower than those of all the other configurations;
- the probability that a layout will remain more efficient than the others: this is the absolute measure of robustness sought.

In fact, a sequence affected by a high probability of efficiency, with respect to all the other configurations, will be a highly robust one, while a configuration with low probability of efficiency will be less flexible to the changes of the external environment.

In other words, the following probability may be calculated:

$$P(y \geq 0) = \int_{-\infty}^{k} g(z)\,\mathrm{d}z = \phi\left(\frac{\overline{M(l)} - M(l^*)}{\sqrt{[\overline{S(l)} + S(l^*)]}}\right) = \vartheta_{\bar{l},l^*}, \tag{30}$$

and it will be:

$$0 \leq \vartheta_{\bar{l},l^*} \leq 1.$$

The probability calculated by equation (30) will be defined as the Layout Configuration Robustness Index (LCRI) of the configuration of layout l^*:

$$LCRI = \phi\left(\frac{\overline{M(l)} - M(l^*)}{\sqrt{[\overline{S(l)} + S(l^*)]}}\right), \tag{31}$$

and, as $0 \leq \vartheta_{\bar{l},l^*} \leq 1$, it must also satisfy:

$$0 \leq LCRI(l) \leq 1.$$

The higher the index, the higher the probability of the layout efficiency and, consequently, the higher the robustness of the configuration.

The adoption of the index proposed reduces the complexity of the calculations required by a simulation-based approach, which is linked to the solution of P layout problems, where P is the number of periods considered. The adoption of the LCRI index decreases the calculation amount by a P^{-1} factor. Furthermore, the index proposed offers a normalized measure of the layout robustness.

In addition, when evaluating the LCRI value for the most robust layout of a stochastic SRMLP, the following theorem must hold.

Theorem: Given a Linear Stochastic Layout Problem, if each flow x_{ij} is described by a normal distribution with mean μ_{ij} and variance σ^2_{ij}, the necessary condition for identifying the mth layout as the most robust solution of a stochastic Single Row Machine Layout Problem is that:

$$LCRI(l) \geq 0.5.$$

Proof: If m is the most robust solution of a stochastic SRMLP, the mean $M(m)$ of the pdf of its $C(m)$ cost function must be the lowest with respect to the averages of the cost functions of the admissible configurations, i.e.

$$M(m) \leq M(l)\, \forall \text{l}.$$

It can also be verified that:

$$M(m) \leq \overline{M(l)}.$$

Once defined:

$$y = \overline{C(l)} - C(m),$$

it is also obtained that:

$$M(y) = \overline{M(l)} - M(m) \geq 0$$

and, according to the notations already introduced:

$$P(y \geq 0) = \int_{-\infty}^{k} g(z)\,\mathrm{d}z = \phi\left(\frac{\overline{M(l)} - M(m)}{\sqrt{[\overline{S(l)} + S(m)]}}\right) = \vartheta_{\bar{l},m} \geq 0.5,$$

thus obtaining that:

$$LCRI(m) \geq 0.5.$$

8. Experimental issues

Let us evaluate the behaviour of the LCRI index for a single stochastic layout problem with five machines. The following set of data is assigned:

M - MATRIX

0	992	830	806	836
992	0	819	1076	820
830	819	0	984	820
806	1076	984	0	908
836	820	820	908	0

S - MATRIX

0	58	266	152	65
58	0	144	146	256
266	144	0	244	212
152	146	244	0	131
65	256	212	131	0

DIMENSION ARRAY 3 4 1 4 2

The index LCRI will be calculated for each one of the (5!)/2 admissible configurations together with the percentage Total Penalty $TP(\%)$. The latter index is evaluated as the percentage deviation with respect to the agile solution, i.e.:

$$TP(\%) = \frac{\sum_{i=1}^{P} C_i^{\mathrm{sol}} - \sum_{i=1}^{P} C_i^{\mathrm{agile}}}{\sum_{i=1}^{P} C_i^{\mathrm{sol}}} \times 100. \tag{32}$$

The graphical representation of the results obtained is presented in figure 6, where each dot represents the result of a layout configuration. The relationship between LCRI and $TP(\%)$ is that low penalty values involve a high LCRI index, i.e. the more robust the configuration, the lower the cost differences with the agile solution. The highest value is found for the most robust solution (5-3-4-1-2), thus showing that LCRI maximization is necessary for the robustness of the optimum layout.

Now, let us consider the LCRI behaviour with respect to different layout problems. This test will allow the appreciation of the robustness of the solutions within an absolute range of values. To this end, a set of different layout problems was considered, each one pertaining to a different class of robustness. In particular, seven stochastic SRMLP, with five machines, were generated, each one associated with a different LPRI value. The LPRI values assigned were 0.99-0.95-0.87-0.85-0.80-0.75-0.65, as shown in figure 7 together with the simulation results.

The trends described in figure 7 allow us to draw some observations:

- the global robustness of the layout problem, defined by the LPRI value, also determines the slope of the curve that expresses the link between the LCRI and the Total Penalty TP(%). As LPRI increases, the slope of the curve increases too; for problems of complete robustness (LPRI = 0.99), the curve is almost vertical. Moreover, few points are found for high values of LCRI (almost equal to 1); on the other hand, while reducing the LPRI value, many points are found for low LCRI values (almost equal to 0), i.e. where the Total Penalty is significant;

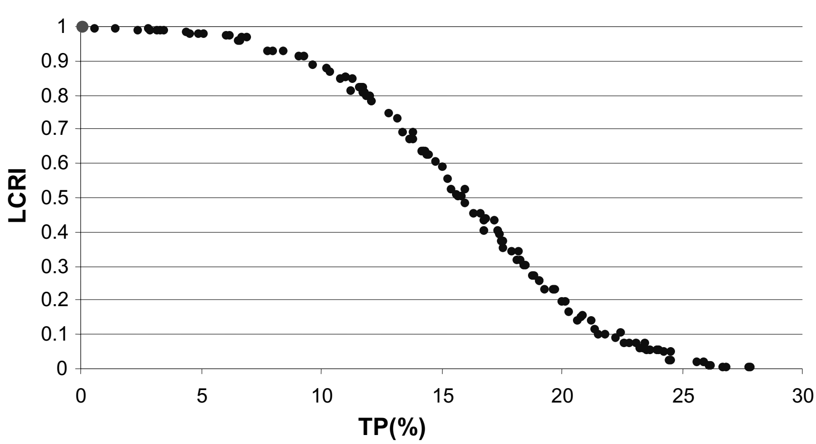


Figure 6. CRI behaviour for a stochastic SRMLP.

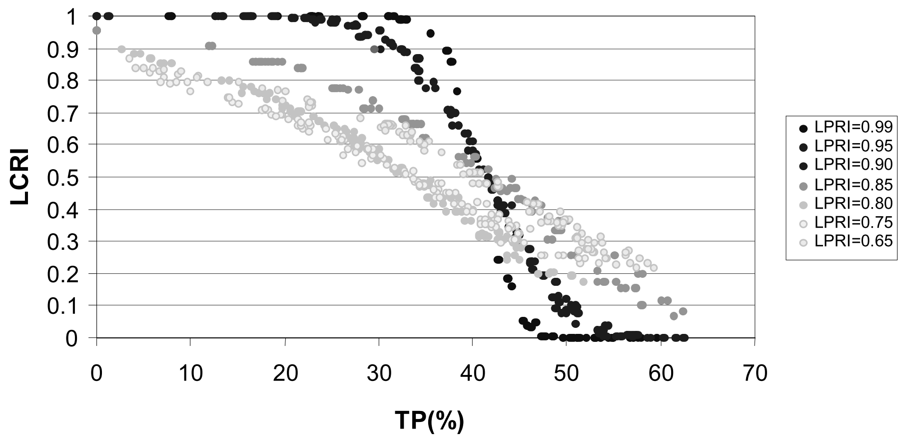


Figure 7. LCRI behaviour for problems with different global robustness.

- as the problem of criticality increases (decreasing of LPRI value), the absolute robustness of the optimal solution decreases. As LPRI decreases, the lower limit of the associated TP(%) value increases. In fact, it varies from values close to 0% (LPRI = 0.99) to 5% (LPRI = 0.65);
- as the global robustness of the problem increases, so does the possibility of optimizing the layout configuration. In fact, critical problems are affected by significant data gathering in the central area, with *TP*(%) values between 20% and 40%.

According to the results presented, a general conclusion may be drawn: the more critical the problem, the smaller the differences in the solutions. On the contrary, robust problems deserve attention for the quality of the procedure adopted in identifying the problem solution; in these cases, there exists a completely robust layout, i.e. the optimum one, and the penalties determined by sub-optimal solutions may be large.

The above considerations are also supported by the results of figure 8: it groups the optimal solutions (robust configuration) of 1000 layout problems, pertaining to

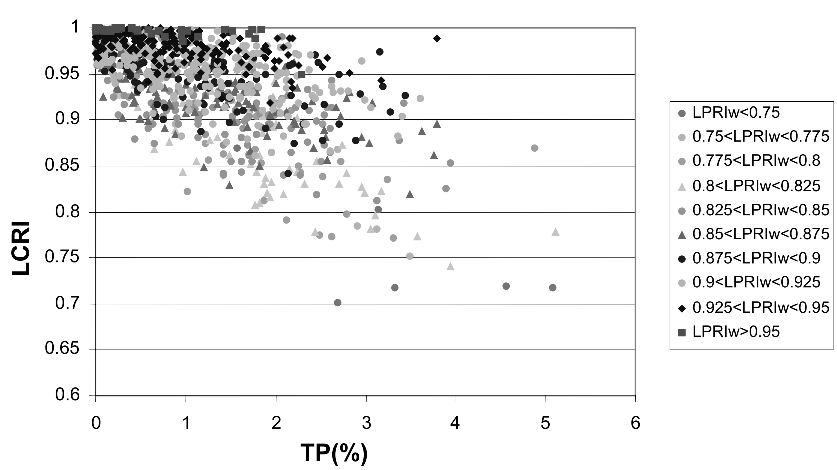


Figure 8. LCRI and penalty values for optimal solutions.

different classes of robustness. For each of these, the LCRI and TP(%) values were calculated and are presented in the graph.

The first experimental confirmation from the graph is that there exists a lower limit for the LCRI value, i.e. LCRI $= 0.5$, as also shown by the proposed theorem. In particular, for the figure 8 problems, the minimum value found for LCRI was 0.7.

Moreover, as already cited in the previous section, it is evident there exists a strong dependency between the width of the range of the LCRI values and the value of the total robustness of the problem (expressed by the LPRI index).

9. Conclusions

This study has discussed the evaluation of the criticality of layout determination in a dynamic environment, i.e. when product demands fluctuate over the time horizon identified. The approach adopted was based on the formulation of indices useful to designers and managers in quantifying the criticality of the layout problem. Consequently, the strategy to be adopted for layout redefinitions may be selected and the level of attention to be paid to the layout configuration is also quantified. To this end, the simulations offered were useful in understanding the practicality of implementing an agile strategy in place of a robust one.

The study also focuses on the mechanisms leading to a degraded performance of a layout, in the case of variations in the productive levels. It is also shown how the robustness of a layout is a problem feature, which depends uniquely on the starting data, as in the case of product demands described by stochastic variables.

Starting from the SRMLP case and a wide set of experimental tests, two analytical indices, LPRI and LCRI, have been proposed in order to estimate, respectively, the robustness of a stochastic layout problem and the robustness of a specific layout configuration for a given problem.

In more detail, the usefulness of these indices is related to their ability to identify the criticality of the layout problem definition. In fact, as in the examples shown, high LPRI indices imply low REL values. The usefulness of the LCRI index emerges when the LPRI is significantly high (i.e. LPRI > 0.9). In such a case, and for a given situation, a layout other than the optimal one may determine consistent economic losses due to material handling (e.g. see figure 7). Therefore, a careful evaluation of the layout design is required.

When external demand is extremely irregular and dynamic, the distribution of the flows may frequently lead to a robust solution operating in degraded conditions. If the alteration in the product demands determines a new layout problem with a significant robustness (LPRI > 0.9), a non-optimal solution will be associated, with relevant cost penalties, as expressed by the relationship between the LCRI and PT(%) values. In similar cases, the redefinition of the layout may be a convenient way out, thus adopting an agile strategy. On the other hand, if the new layout problem is described by low robustness indices (LPRI < 0.8), there is a reduced risk of performance deterioration, as problem solutions are generally undifferentiated and a layout redefinition may be economically unjustified. In such a case, it has been shown that the LCRI index is almost constant for the problem given.

The theory proposed is based on the analysis of the stochastic nature of the flow matrix and it seems to be promising for application to layouts other than the single row one (e.g. loop layout and QAP). In fact, the starting point adopted (i.e. the flow matrix) is generally valid for the whole set of layout types. The above considerations show how various developments of the present study are foreseeable in the near future. Some additional research guidelines, and where the authors will be involved, are:

- the study and evaluation of other typologies of layout, to appreciate and compare their performances and introduce the necessary modifications to the indices proposed for the SRMLP case;
- the analysis of other types of production fluctuations, e.g. those due to the introduction of new products and to the flexibility of routing;
- the utilization of the approach proposed in a real industrial case.

Acknowledgements

The authors are indebted to the referees for their constructive comments, which enabled the improvement of the quality of the present study. This work was partially supported by MIUR funds.

References

Afentakis, P., Millen, R. A. and Solomon, M. N., 1990, Dynamic layout strategies for Flexible Manufacturing Systems. *International Journal of Production Research,* **28**, 311–323.

Benjaafar, S. and Sheikhzadeh, M., 2000, Design of flexible plant layouts. *IIE Transactions,* **32**, 309–322.

Hassan, M. M., 1994, Machine layout problem in modern manufacturing facilities. *International Journal of Production Research,* **32**, 2559–2584.

Kochhar, J. S. and Heragu, S. S., 1999, Facility layout design in a changing environment. *International Journal of Production Research,* **37**, 2429–2446.

Kouvelis, P. and Kiran, A. S., 1991, Single and multiple period layout models for automated manufacturing systems. *European Journal of Operational Research,* **52**, 397–412.

Law, A. and Kelton, D., 1994, *Simulation Modeling and Analysis* (New York: McGraw-Hill).

Lin, C. L. and Sharp, G. P. 1999, Quantitative and qualitative indices for the plant layout evaluation problem. *European Journal of Operational Research,* **116**, 100–117.

Malmborg, C. J., 1999, Empirical approximation of volume distance distributions for line layout problems. *International Journal of Production Research,* **37**, 375–392.

Malmborg, C. J. and Bukhary, F. A., 1997, Material flow analysis and volume distance sampling in heuristic line layout procedures. *International Journal of Production Research,* **35**, 2045–2063.

Malmborg, C. J., Bukhary, F. A. and Dermott, C., 1998, Predicting volume distance performance for layout problems with static distance functions. *International Journal of Production Research,* **36**, 2339–2354.

Meller, D. and Gau, K. Y., 1996, Facility layout objective functions and robust layouts. *International Journal of Production Research,* **34**, 2727–2742.

Rajasekharan, M., Peters, B. A. and Yang, T., 1998, A genetic algorithm for facility layout design in flexible manufacturing systems. *International Journal of Production Research,* **36**, 95–110.

Rosenblatt, M. J. and Lee, H. L., 1987, A robustness approach to facilities design. *International Journal of Production Research,* **25**, 479–486.

Rosenblatt, M. J. and Kropp, D. H., 1992, The single period Stochastic Layout Problem. *IIE Transactions,* **24**, 169–176.

Tompkins, J. A. and White, J. A., 1996, *Facilities Planning* (New York: Wiley).